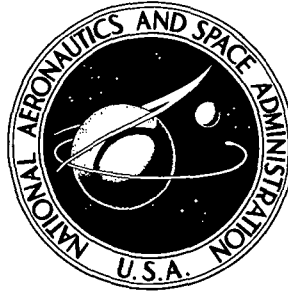


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**A CURVATURE-CORRECTED KIRCHOFF  
FORMULATION FOR RADAR SEA-RETURN  
FROM THE NEAR VERTICAL**

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## Summary

A new theoretical treatment of the problem of electromagnetic wave scattering from a randomly-rough surface is given. A high frequency correction to the Kirchhoff approximation is derived from a field integral equation for a perfectly conducting surface. The correction is of the form  $\underline{J}^{(1)} = \underline{S} \underline{J}^{(0)}$  where  $\underline{J}^{(0)}$  is the Kirchhoff value of the current density, and  $\underline{S}$  is a linear function of the second derivatives of surface height. The correction is seen to be identical to an asymptotic form found by Fock (1945) in his investigation of diffraction by a convex paraboloid.

The corrected current density is substituted into the far-field Stratton-Chu integral, and average backscattered powers for the four linear polarization combinations are computed on the assumption that the scattering surface is describable as a homogeneous Gaussian random process.

The strength of the solution is that local diffraction effects (arising from surface curvature) are properly correlated with surface height and slope without requiring their smallness.

Application to radar backscatter and natural microwave emission from the sea is discussed. It is concluded that this "corrected Kirchhoff" formulation offers a superior predictive capacity for co-polarized and depolarized returns from the near vertical,  $\theta < 45^\circ$ . Extension to the bistatic case is recommended for application to the natural emission problem. The evolution of a new "composite" model of backscatter combining this solution with small perturbation results suggests itself.

## I. Introduction

### 1.1 Origins

A satellite-borne combination radar-radiometer has been proposed as a remote sensing device for monitoring wind field/wave field conditions over the world ocean (see, for a recent version of this proposal, Moore and Pierson, 1971). This proposal has stimulated much of the recent theoretical and experimental work on radar backscatter and natural microwave emission from a wind-roughened sea. In particular, the research reported here was motivated by a need for a broader theoretical basis for understanding and predicting microwave scattering by the sea. A good part of this research has been previously documented in a New York University technical report (Jackson, 1971).

### 1.2 Rough surface scattering theory and radar sea-return

The general problem of electromagnetic-wave scattering from a plane rough surface has been approached in a variety of ways. Some authors have dealt with exact solutions for scattering from certain simple (idealized) surfaces such as a surface composed of periodic rectangular corrugations (Deryugin, 1960). Exact solutions for arbitrary surfaces can be obtained by the numerical solution of an integral equation for the surface field (Lenz, 1971). Except for one-dimensional surfaces possessing a small amount of structure (e.g., a small number of "hills" and "valleys") the computational time is prohibitive. Many analytical methods have been developed for the class of surfaces which can be called slightly rough, defined

as having small slopes and heights small compared to the wavelength. Exact solutions for deterministic slightly rough surfaces can be obtained by Rayleigh's method, or Meecham's (1956) variational technique. Rayleigh's method has been randomized by Rice (1951), and this provides one of the most powerful methods for handling scattering from randomly rough surfaces. Twersky (1957) has used a Rayleigh image method for computing exactly the scattering by a random array of "bosses" or protuberances on a plane. Exact methods--while impractical for application to randomly rough surfaces with a high degree of "structure"--can be very useful in the testing of approximate methods developed for general classes of randomly rough surfaces.

Some authors have taken conceptual approaches which are at the same time simple and instructive --for example, Katzin's (1957) slope-facet model and Long's (1965) dipole model of sea return. Katzin's model is interesting, for it entertains an important property of radar sea-return, that near vertical incidence (radar pointing downward) the backscatter mechanism is dominantly specular reflection, while backscatter from large angles of incidence is controlled by diffraction processes. Katzin's model has been extended by Rouse (1970).

Of approximate methods which have found practical application to scattering from continuously distributed random rough surfaces, there are essentially three:

- a) Geometrical optics
- b) Physical optics (Kirchoff theory)

c) Small perturbation (Rayleigh-Rice theory).

Geometrical optics (or ray optics) formulations, because of their great simplification of the electromagnetic problem, are capable of handling such phenomena as shadowing and multiple scattering (Lynch and Wagner, 1970). In considering the generally mild slopes of ocean surface waves, a ray-optical type of multiple scattering is not likely to be a significant part of the backscatter mechanism. In the general bistatic case, however, the process of multiple reflections may be important. Shadowing effects can similarly be ignored for angles removed from grazing incidence. Shadowing is to a first approximation a simple enough process that it can be included in a physical optics formulation (Sancer, 1969).

Most of the work in the last decade has been based upon either (or both) physical optics (randomized Kirchhoff method) or methods of small perturbation. The randomized Kirchhoff method developed by Beckmann (Beckmann and Spizzichino, 1963, Chs. 3 and 5) and others uses the physical optics (Helmholtz) integral with the so-called Kirchhoff or tangent-plane approximation to the boundary values of the field. The Kirchhoff method is good for softly undulating surfaces having everywhere a local radius of curvature large compared to the electromagnetic wavelength. An advantage of the Kirchhoff method (over small perturbation methods) is that surface height variations do not necessarily have to be small compared to the radar wavelength. A shortcoming of Kirchhoff theory is that with its tangent-plane approximation it cannot account for polarization effects. Apart from its inability to account for



polarization effects, Kirchoff theory has suffered because its inherent strength was not exploited. Often, what amounts to a stationary phase approximation to the Kirchoff integral is made. The stationary phase approximation is equivalent to geometrical optics; so, the ability to account for diffraction effects is lost. Chia (1968), in applying Kirchoff theory to radar sea-return, appears to be the first to have avoided this approximation by using a realistic wave-height covariance function in the Kirchoff integral.

Small perturbation theory has gained increasing favor in the last few years among scientists working on radar sea-return. This is primarily because of its ability to account for polarization effects but also because of its simplicity. The explicit dependence on the wave-height spectrum pointed the way to using a realistic representation of the rough sea surface (Valenzuela, 1968; Wright, 1968). Small perturbation methods have been developed by several authors (Bass, 1961; Wright, 1966), notably by Rice (1951). Rice's randomized Rayleigh method seems to be the superior, for it is capable of iteration to higher order in height and slope, and is capable of predicting depolarization in the plane of incidence (Valenzuela, 1967, 1968). The Rayleigh-Rice method--although having some commonality with the Kirchoff method--is a fundamentally different approach to the scattering problem; whereas Kirchoff theory is a high frequency treatment, Rice's theory is a low frequency method. In the high frequency approximation, the field in the vicinity of a surface point is dependent only on the local geometry of the surface, i. e., height and slope. In the Rayleigh approach, the field at a point is related to integral properties (rather than local properties) of the

surface. This local versus integral (or modal) duality in electromagnetic theory is discussed by Felsen (1964) in his review of high frequency diffraction.

### 1.3 Outline

The theoretical approach taken in this work is a high-frequency one, and is essentially an extension of the randomized Kirchhoff method. A curvature correction to the Kirchhoff approximation is derived from a field integral equation, and the corrected boundary values are used in the Stratton-Chu far-field integral (vector form of the Helmholtz integral).

In the following section, Beckmann's development of scalar Kirchhoff theory is recapitulated in order to place the theory of Sections V and VI in its proper perspective. Section III contains a discussion of the Kirchhoff solution, and presents the results of Kirchhoff theory applied to a scattering surface with a  $K^{-3}$  spectral law. Section IV presents the results of small perturbation theory. Because of the complexity of the scattering integrals arrived at and their requirement of a detailed knowledge of the wave-height covariance function, no thorough computation and comparison with sea-return data is made. However, some sample calculations are given, the nature of the solution is discussed and estimates of its strength are made.

## II. The Randomized Kirchhoff Method According to Beckmann\*

Consider the scattering situation depicted in Fig. 1. A monochromatic source (radar) illuminates a portion  $S_0$  of a rough conducting surface. Beckmann deals with the scalar electric field  $E$  scattered by the surface. The surface field vanishes outside of  $S_0$ , and the scattered field satisfies the radiation condition at infinity. The electric field scattered toward an observation point  $P$  is given by the Helmholtz integral involving the (unknown) field boundary values:

$$E(P) = \frac{1}{4\pi} \int_{S_0} \left( E \frac{\partial G}{\partial n} - G \frac{\partial E}{\partial n} \right) dS \quad (2.1)$$

where  $E$  is the total electric field on the surface;  $\partial/\partial n$  stands for the normal derivative directed outwards (upwards) from the conducting volume; and  $G$  is the Green's function for (three-dimensional) free space,

$$G = \frac{e^{-ikr}}{r}$$

where  $k$  is the microwave propagation constant and  $r$  is the distance from source points on the surface to the observation point. If  $P$  is in the far-field (Fraunhofer zone)  $r$  can be

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\*In Beckmann and Spizzichino (1963), Chapters 3 and 5.

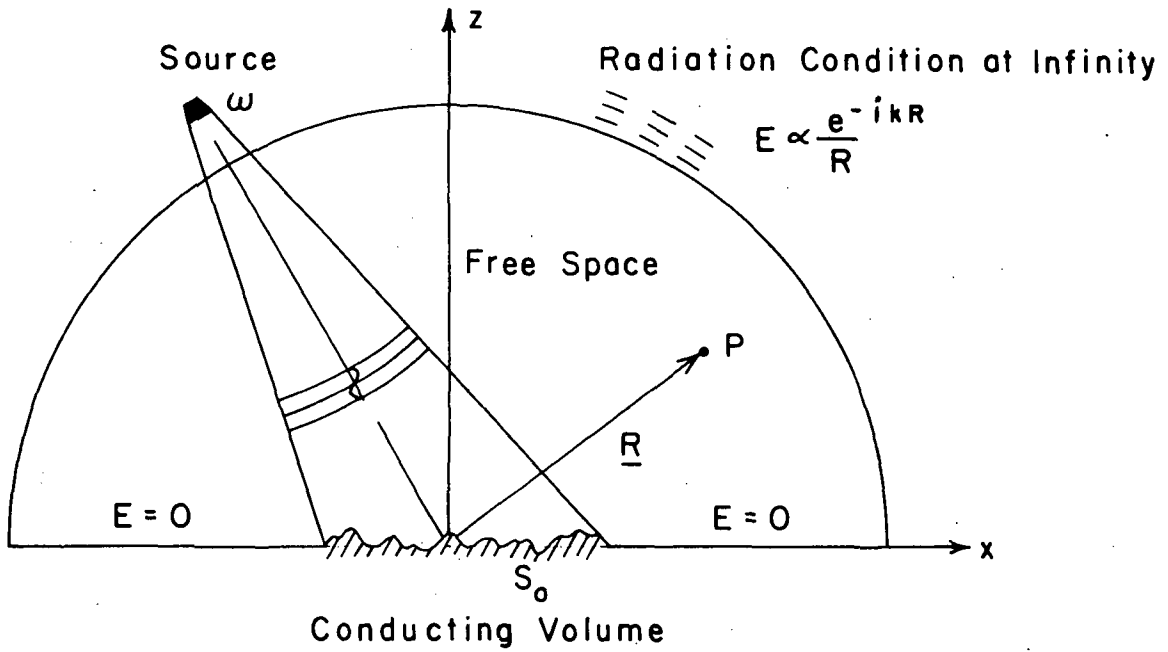


Fig. 1. The scattering situation.

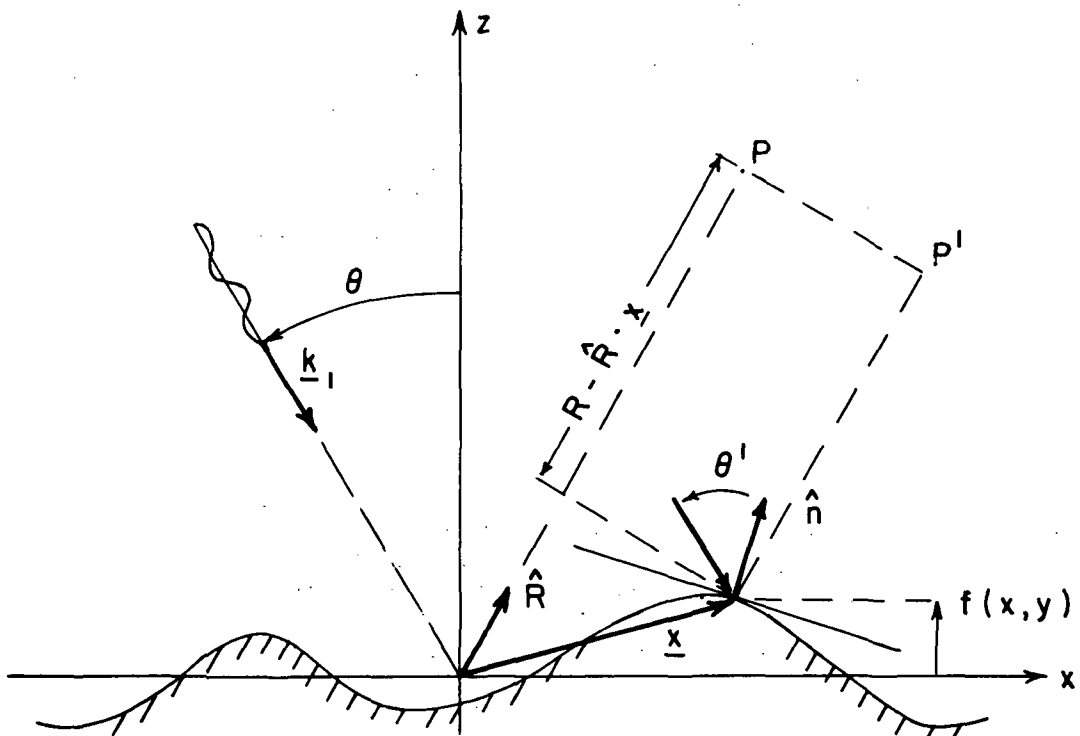


Fig. 2. Defining geometry and notation.



where  $\hat{n}$  is the unit outward surface normal.

The incident field  $E^i$  is taken to be a plane wave\* of unit amplitude. With the time-harmonic dependence  $e^{i\omega t}$  suppressed,  $E^i$  is written as the phasor,

$$E^i = e^{-i\mathbf{k}_1 \cdot \mathbf{x}} \quad (2.5)$$

where  $\mathbf{k}_1$  is the propagation vector of the incident wave.

The Kirchoff approximation to the field boundary values  $E$  and  $\partial E / \partial n$  consists in assuming that the field in the vicinity of (an "epsilon" neighborhood) and at a point on the surface is nearly equal to the field which would exist on an infinite tangent plane at the point. This is a type of "high frequency" approximation. For a high enough microwave frequency (wave number), the surface curvature "appears" mild to the radiation and the surface can be considered to be locally flat. There is then a perfect reflection of the incident wave in accordance with the geometrical optics Law of Reflection; the amplitude and phase of the reflected wave are given by Fresnel's formulas. Then, writing the field in the vicinity of the surface as an incident wave plus a reflected wave, we have for the total field and its normal derivative evaluated on the surface:

$$E = (1 + R)E^i \quad (2.6a)$$

$$\frac{\partial E}{\partial n} = -i\mathbf{k}_1 \cdot \hat{n}(1 - R)E^i. ** \quad (2.6b)$$

---

\* The justification for ignoring the sphericity of an incident wave radiated by a point source (radar) follows the same arguments given for the far-zone approximation on the scattered field (see previous footnote).

\*\*In writing (2.6) with the incident wave  $E^i$  given by eq. (2.5), it is tacitly assumed that there is no multiple scattering nor shadowing.

The Fresnel reflection coefficient  $\mathcal{R}$  is in general a matrix. Treating  $\mathcal{R}$  as a scalar coefficient might seem to be of dubious validity, but it will serve us in a formal development.

Brekhovskikh\* has shown that the Kirchoff approximation is valid if

$$4\pi r_c \cos \theta' \gg \lambda \quad (2.7a)$$

where  $r_c$  is the radius of curvature,  $\theta'$  is the "local" angle of incidence (the angle included between the local normal  $\hat{n}$  and the incident ray), and  $\lambda$  is the radar wavelength. Wait and Conda\*\* have given the criterion,

$$\pi r_c \cos^3 \theta' \gg \lambda. \quad (2.7b)$$

These inequalities should not be interpreted in a strict sense, since large third derivatives can exist even if the second derivatives are small. We should really interpret  $r_c$  as a root-mean-square value for the surface, giving an indication of the degree of "smoothness".

Substituting eqs. (2.4-2.6) into the Helmholtz integral (2.1), we get for the far-zone scattered field:

$$E = \frac{e^{-ikR}}{4\pi R} \int_{S_0} [ik\hat{R} \cdot \hat{n}(1 + \mathcal{R}) + i\underline{k}_1 \cdot \hat{n}(1 - \mathcal{R})] e^{i(k\hat{R} - \underline{k}_1) \cdot \underline{x}} dS \quad (2.8)$$

Departing from Beckmann's development, we specialize the problem to the monostatic case (backscatter). The observation

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\*See Beckmann and Spizzichino (1963), p. 29.

\*\*Ibid., p. 29.

point then coincides with the source, and the incident propagation vector can be written as  $\underline{k}_1 = -k\hat{R}$ . Eq. (2.8) becomes:

$$\underline{E} = \frac{ike^{-ikR}}{4\pi R} \int_{S_0} 2\hat{R} \cdot \hat{n} e^{i2k\hat{R} \cdot \underline{x}} dS \quad (2.9)$$

We adopt the following notation (Fig. 2): Let the surface be described in the  $(x, y, z)$  Cartesian system by

$$z = f(x, y) \quad .$$

The coordinate vector of surface points is then

$$\underline{x} = (x, y, f) \quad .$$

The normal vector is given by

$$\hat{n} = (-f_x, -f_y, 1) \cos \omega \quad ,$$

$$\cos \omega = (1 + f_x^2 + f_y^2)^{-\frac{1}{2}}$$

where  $f_x$  and  $f_y$  are partial derivatives. The surface area element is given by

$$dS = \sec \omega \, dx \, dy \quad .$$

The plane of incidence is formed by  $\hat{R}$  and the  $z$ -axis. The  $x$  and  $y$  axes are oriented so that the  $x$ -axis lies in the plane of incidence. The angle of incidence  $\theta$  is measured positive where the incident ray comes from the left (negative  $x$ -direction). The unit vector  $\hat{R}$  is given by

$$\hat{R} = (-\alpha, 0, \gamma)$$

where

$$\alpha = \sin \theta$$

$$\gamma = \cos \theta \quad .$$

Equation (2.9) becomes



$$E = \frac{ike^{-ikR}}{4\pi R} \iint_{A_0} 2\mathcal{R} \cdot (\gamma + af_x) \cdot e^{-i2k(ax-\gamma f)} dx dy \quad (2.10)$$

where  $A_0$  is the (horizontal) illuminated area. Now assume for the moment that the surface is perfectly conducting. Then,  $\mathcal{R} = \pm 1$  (the sign depending on polarization) and  $\mathcal{R}$  can be treated as a constant and removed from under the integral.\* With  $\mathcal{R} = \pm 1$ , (2.10) is integrated by parts to yield

$$E = \pm \frac{ike^{-ikR}}{4\pi R} \cdot \frac{2}{\gamma} \iint_{A_0} e^{-i2k(ax-\gamma f)} dx dy + \text{"edge terms"}. \quad (2.11)$$

For large areas  $k^2 A_0 \gg 1$ , the edge terms are negligible. This is not obvious from the form of (2.11); but, it is physically reasonable that--provided the surface is moderately rough and grazing incidence is avoided--edge effects are unimportant in the scattering problem. The average backscattered power is proportional to  $\langle |E|^2 \rangle$  where the brackets denote expectation, or ensemble average. Neglecting the edge terms, we form  $|E|^2 = EE^*$  as a two-fold integral over  $A_0$ , and take the average over all realizations of the surface:

$$\langle |E|^2 \rangle = \left( \frac{k}{2\pi R \gamma} \right)^2 \iiint \langle e^{i2k\gamma(f' - f)} \rangle e^{-i2k\gamma(x' - x)} dx' dy' dx dy \quad (2.12)$$

The expectation

$$\phi(2k\gamma, -2k\gamma) = \langle e^{i2k\gamma(f' - f)} \rangle \quad (2.13)$$

is the two-dimensional characteristic function of the random vector  $(f', f)$  evaluated at  $(2k\gamma, -2k\gamma)$ . If  $f$  is a stationary (homogeneous)

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\*This is not immediately evident. It turns out, however, that the vector formulation of the Kirchhoff integral for perfect conductivity yields precisely (2.10) with the effect of polarization properly accounted for by the  $\pm$  sign in front of the integral. (Cf. eqs. 6.2a, d)

Gaussian random process of zero mean, then

$$\phi = e^{-4k^2 \gamma^2 B(\underline{\xi})} \quad (2.14)$$

where

$$B(\underline{\xi}) = -R(0) + R(\underline{\xi}) \quad (2.15)$$

and  $R$  is the surface height covariance function

$$R(\underline{\xi}) = \langle f' f \rangle \quad (2.16)$$

and  $\underline{\xi}$  is the "lag" or "separation" vector,

$$\underline{\xi} = (\xi, \eta) = (x' - x, y' - y) \quad (2.17)$$

In terms of the surface height variance,  $\sigma^2 = R(0)$ , and the auto-correlation coefficient,  $\rho = R(\underline{\xi})/\sigma^2$ ,  $B$  can be written as

$$B(\underline{\xi}) = -\sigma^2 + R(\underline{\xi}) = -\sigma^2(1 - \rho) \quad (2.18)$$

If  $A_0$  is large compared to the scale of roughness correlation lengths" the double-area integral (2.12) is nearly equal to

$$\langle |E|^2 \rangle = \left( \frac{k}{2\pi R \gamma} \right)^2 A_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-4k^2 \gamma^2 B} e^{-i2ka\xi} d\xi d\eta \quad (2.19)$$

The infinite limits on the  $\eta$  integration are not to be taken in a strict sense. For large  $\eta$  the exponential approaches the constant asymptotic value  $\exp\{-4k^2 \gamma^2 \sigma^2\}$ . The integral over  $\eta$  then increases linearly with  $\eta$ , and is independent of  $\xi$ . If the infinite  $\eta$  limits are interpreted in the sense of the limit

$$\lim_{Y \rightarrow \infty} \int_{-\infty}^{\infty} \int_{-Y}^Y e^{4k^2 Y^2 B} e^{-i2ka\xi} d\xi d\eta$$

then it is seen that (except for vertical incidence,  $a = 0$ ) the phasor nullifies the constant contribution from large  $\eta$  and the integral exists. In practice we generally deal with very rough surfaces for which  $4k^2 Y^2 \sigma^2 \gg 1$  and the exponential for large lag is extremely small. A numerical integration over  $\eta$  would then be stopped at some prescribed value of  $\eta$  for which the exponential is very small.

Since the covariance function  $R(\xi)$  is symmetric about the origin, (2.19) simplifies to

$$\langle |E|^2 \rangle = \frac{1}{2} \left( \frac{k}{\pi R Y} \right)^2 A_0 \int_0^{\infty} d\xi \cos 2ka\xi \int_{-\infty}^{\infty} e^{4k^2 Y^2 B} d\eta. \quad (2.20)$$

The backscattered power is conventionally given in terms of the normalized isotropic radar cross section,  $\sigma^\circ$ , defined by

$$\sigma^\circ = \frac{4\pi R^2}{A_0} \langle |E|^2 \rangle. \quad (2.21)$$

Then, (2.20) becomes

$$\sigma^\circ = \frac{2k^2}{\pi} \cdot \frac{1}{2} \int_{-Y}^Y \int_0^{\infty} e^{4k^2 Y^2 B} \cos 2ka\xi d\xi d\eta. \quad (2.22)$$

### III. On the Nature of the Kirchoff Solution. Radar Sea-Return

The Kirchoff integral (2.22) contains information on the two backscatter regimes: the "specular" (ray-optic) regime which preponderates for small angles of incidence, and the "small perturbation" (diffraction) regime which accounts for the backscatter from large angles of incidence.

#### 3.1 The stationary phase approximation

The high frequency limit of physical optics yields geometrical optics: symbolically,

$$\lim_{k \rightarrow \infty} (\text{physical optics}) = \text{geometrical optics}$$

The high frequency limit is equivalent to a case where (1) the surface is very smooth, and (2) the phase modulation is very deep. The "deep fade" condition (Hagfors, 1964) means we must have (for backscatter):

$$4k^2 \gamma^2 \sigma^2 \gg 1. \quad (3.1)$$

In the high frequency case characterized by (3.1), the exponential  $\exp\{4k^2 \gamma^2 B\}$  becomes negligible outside of a small region about the origin. The meaning of the smoothness condition is that within this "neighborhood" of the origin, the covariance function can be represented by a Taylor series truncated at second degree. Thus, we can write  $B(\underline{\xi})$  as the paraboloid:

$$B = R_{xx}(\underline{0}) \frac{\xi^2}{2} + R_{xy}(\underline{0}) \xi \eta + R_{yy}(\underline{0}) \frac{\eta^2}{2}$$

Considering (for simplicity) the special case where  $\xi$  and  $\eta$  are the

principal axes of the ellipse  $B = \text{constant}$ ,  $B$  can be written as

$$B = -\frac{1}{2} \sigma_x^2 \xi^2 - \frac{1}{2} \sigma_y^2 \eta^2 \quad (3.2)$$

where  $\sigma_x^2 = -R_{xx}(0)$  and  $\sigma_y^2 = -R_{yy}(0)$  are the slope variances in the  $x$  and  $y$  directions. The rms slope  $\sigma_s$  is invariant with respect to coordinate rotation,

$$\sigma_s = \sqrt{\sigma_x^2 + \sigma_y^2} \quad (3.3)$$

Putting (3.2) into (2.22) and integrating, we find (letting  $\sigma_1^\circ$  represent the high frequency limit):

$$\sigma_1^\circ = \frac{1}{2\sigma_x\sigma_y} e^{-\frac{1}{2} \frac{\tan^2 \theta}{\sigma_x^2}} \quad (3.4)$$

The cross-section (3.4) is independent of  $k$ , consistent with the fact that it is the high frequency limit--geometrical optics. Another way of seeing that (3.4) is a geometrical optics limit is to note the  $\sigma^\circ$  is simply proportional to the probability of a surface facet satisfying the specular (ray-optic) condition for backscatter:  $f_x = \tan \theta$ .

For an isotropic surface,  $\sigma_x = \sigma_y$ , and (3.4) becomes

$$\sigma_1^\circ = \frac{1}{\sigma_s^2} e^{-\frac{\tan^2 \theta}{\sigma_s^2}} \quad (3.5)$$

For reference, we should like to know the cross-section for a two-dimensional scattering situation. One has to go back to the Helmholtz integral for two dimensions:

$$E = \frac{1}{4i} \int_{S_0} \left( E \frac{\partial G}{\partial n} - G \frac{\partial E}{\partial n} \right) dS .$$

where  $G$  is the zeroth-order Hankel function of the second kind:

$$G = H_0^{(2)}(kr) .$$

Making the far-zone approximation and following a procedure identical to the three-dimensional case, but defining the cross-section for cylindrical waves as

$$\sigma^\circ = \frac{2\pi R}{X} \langle |E|^2 \rangle \quad (3.6)$$

where  $X$  is the linear extent of illumination, we get:

$$\sigma^\circ = \frac{2k}{\gamma} \int_0^\infty e^{-4k^2 \gamma^2 B} \cos 2ka\xi \, d\xi \quad (3.7)$$

The high-frequency limit of (3.7) is:

$$\sigma_1^\circ = \frac{\sqrt{\pi/2}}{\sigma_x \gamma} e^{-\frac{1}{2} \frac{\tan^2 \theta}{\sigma_x^2}} \quad (3.8)$$

Taking the high frequency limit of the cross-section (2.22) is seen to be equivalent to making a "stationary phase" approximation\* to the field integral (2.10). The phase  $\phi = 2k(ax - \gamma f)$  is "stationary" for the specular points having slope  $f_x = a/\gamma = \tan \theta$  and  $f_y = 0$ . The stationary phase approximation provides a (mathematical) rationale for accounting for finite conductivity effects in the specular

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\*For example, see Lamb, Sir H.: Hydrodynamics. Dover Publications, New York, 1963, p. 395.

(near vertical) regime. Referring again to eq. (2.10), we see that  $\mathcal{R}$  is brought out of the integral and evaluated at the local angle of incidence  $\theta' = 0$  for stationary or specular points. This is just the mathematical version of what we should expect on physical grounds: that where specular reflection is the dominant mode of scattering, the backscattered field is composed primarily of incident waves reflected at locally vertical incidence. Thus, the effect of finite conductivity is to a first approximation (a very good first approximation for radar return from the sea) simply to reduce the backscattered signal by a factor of  $|\mathcal{R}(0)|^2$ . And this is true regardless of polarization, since at (locally) vertical incidence the distinction between horizontal and vertical polarization\* disappears.

Stogryn (1967a) came to this conclusion using a vector Kirchhoff formulation and making the "stationary phase" approximation. Kaufman (1971)--apparently unaware of Stogryn's conclusions--used a vector Kirchhoff formulation for finite conductivity, and computed cross sections for different polarizations. Unfortunately, Kaufman made the stationary phase approximation, so that no information was gained on the effects of finite conductivity (as they manifest in a Kirchhoff formulation) in the diffraction regime at larger incidence angles. For reference, let us write down our conclusion symbolically as

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\*These terms have not yet been defined. Vertical polarization (V) means the E-vector is in the plane of incidence (mean plane or local plane); horizontal polarization (H) means the E-vector is perpendicular to the plane of incidence, that is, lies in the plane of the surface (mean or local).

$$\sigma_1^\circ = |\mathcal{R}(0)|^2 \sigma_1^\circ(\infty) \quad (3.9)$$

where  $\sigma^\circ(\infty)$  stands for the cross-section of a perfectly conducting surface.

### 3.2 The "small perturbation" approximation

Small perturbation theory has been remarkably successful in predicting sea-return from large incidence angles (e.g., see Guinard et al., 1971). The reason is that toward larger angles of incidence a larger portion of the surface becomes effectively small amplitude by the Rayleigh criterion. The Rayleigh criterion is actually contained in the Kirchoff formulation as the exponent  $4k^2 \gamma^2 \sigma^2$  (cf. Beckmann and Spizzichino, 1963, p. 93). A surface is smooth by the Rayleigh criterion if

$$2k\gamma\sigma \ll 1. \quad (3.10)$$

Although the sea-surface variance is generally several orders of magnitude too large to satisfy (3.10), what happens is that at larger angles, the exponent  $4k^2 \gamma^2 B$  remains small over larger lags. Also, the horizontal wave number  $2k\alpha$  increases, causing relatively more energy to be contributed (to the Kirchoff integral) from the small lag neighborhood where the exponent is small.

Assuming that (3.10) is satisfied, we can approximate the exponential by its first-order expansion,

$$e^{4k^2 \gamma^2 B} \sim 1 - 4k^2 \gamma^2 \sigma^2 + 4k^2 \gamma^2 R(\underline{\xi}) \quad (3.11)$$



The Kirchoff integral (2.22) then becomes, approximately (naming the small perturbation cross-section  $\sigma_2$ ):

$$\sigma_2^\circ = \frac{8k^4}{\pi} \int_0^\infty \int_{-\infty}^\infty R(\underline{\xi}) \cos 2k\alpha \xi \, d\xi$$

+ a specular term at  $\theta = 0$  (a Dirac spike) .

From the definition of the wave-height (roughness-amplitude) spectrum (half-spaced):

$$S(\underline{\ell}) = \frac{1}{2} \int_0^\infty \int_{-\infty}^\infty R(\underline{\xi}) \cos(\underline{\ell} \cdot \underline{\xi}) \, d\xi \, d\eta \quad (3.12)$$

we see that  $\sigma_2^\circ$  can be expressed simply as

$$\sigma_2^\circ = 8\pi k^4 S(2k\alpha, 0) . \quad (3.13)$$

The wave number  $\underline{\ell} = (2k\alpha, 0)$  is referred to as the Bragg back-scatter condition (Wright, 1968). The physical meaning is understood as follows: Huygens wavelets are in phase in the backscatter direction when the source elements are located a distance

$$L = \frac{\lambda}{2} \csc \theta$$

apart. And we have  $\ell = 2\pi/L = 2k\alpha$ .

We should also like to know the two-dimensional form of  $\sigma_2^\circ$ . If the (one-sided) one-dimensional spectrum is defined as

$$S(\ell) = \frac{2}{\pi} \int_0^\infty R(\xi) \cos \ell \xi \, d\xi , \quad (3.14)$$

then applying (3.11) to (3.7) we get:

$$\sigma_2^\circ = 4k^3 \pi S(2k\alpha) . \quad (3.15)$$

### 3.3 Radar sea-return. The idea of a composite surface.

Chia (1968) appears to be the first to have used a realistic wave height covariance function in a Kirchoff formulation. What Chia did, essentially, was to take the equilibrium range spectral law (Phillips, 1966) and cosine transform to find the covariance function. The equilibrium range spectral law is (in polar coordinates):

$$S(K, \varphi) = AK^{-4} F(\varphi) \quad (3.16)$$

where  $A$  is a universal constant and  $F$  is a dimensionless spreading factor normalized so that

$$\int_0^{2\pi} F(\varphi) d\varphi = 1.$$

The equilibrium range is supposed to exist in a fully aroused sea between wave numbers near the spectral peak down to wave numbers approaching the capillary-wave regime. Assume that isotropic conditions prevail, and that the spectrum can be defined by

$$S(K) = \frac{A}{\pi} K^{-4}, \quad \begin{cases} |\varphi| \leq \pi \\ K_0 \leq K < \infty \end{cases} \quad (3.17)$$

where  $K_0$  is a low wave number near the spectral peak,

$$K_0 \sim g/U^2; \quad (3.18)$$

where  $g$  is the acceleration of gravity and  $U$  is the wind speed at a nominal anemometer height.

For microwaves, the propagation constant  $k$  is several orders of magnitude larger than  $K_0$ . Scattering is determined primarily by the wave structure in a wave number domain centered about

the Bragg backscatter condition,  $K = 2ka$ . This allows a simple approximation to the covariance function corresponding to (3.17). For small

lags  $K_0 r \ll 1$  ( $r = \sqrt{\xi^2 + \eta^2}$ ):

$$R(r) = \frac{A}{\pi} \int_{K_0}^{\infty} \int_{-\pi/2}^{\pi/2} K^{-3} \cos(Kr \cos \varphi) dK d\varphi$$

$$\sim \sigma^2 + A \frac{r^2}{4} (-1 + \gamma + \ln K_0 r/2).^* \quad (3.19)$$

where  $\gamma = 0.577\dots$  is Euler's number.

Since the high-frequency portion of the wave spectrum is always nearly isotropic, a one-dimensional counterpart to (3.16) can only be a fictitious analog. However, if one imagines all the wave energy to be concentrated into one direction, then  $F = \delta(\varphi)$ , a Dirac spike, and the one-dimensional spectrum becomes

$$S(K) = AK^{-3} \quad (3.20)$$

The approximation to the covariance function for small lags,

$K_0 \xi \ll 1$ , is

$$R(\xi) = A \int_{K_0}^{\infty} K^{-3} \cos K\xi dK$$

$$\sim \sigma^2 + A \frac{\xi^2}{2} \left( -\frac{3}{2} + \gamma + \ln K_0 \xi \right) \quad (3.21)$$

It is possible that funnelling all the wave energy into one direction might produce an unrealistically large amount of scattering (i. e., in the two-dimensional approximation to the three-dimensional scattering problem). Let us allow for a variation of the spectral constant  $A$  by imagining that only a certain band of directions  $\Delta\Phi$

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\*Luke, Y. L., Integrals of Bessel Functions, McGraw-Hill, 1962, p. 48.

are funnelled into the one direction, that is, let  $A \rightarrow \frac{\Delta\Phi}{\pi} A$ . If nothing else, this artifice will let us see the variation of  $\sigma^\circ$  with a changing spectral constant.\*

Figure 3 shows the two-dimensional Kirchoff solution (eq. (3.7)) using the covariance function given by eq. (3.21). Nominal values of  $A = 5 \times 10^{-3}$ ,  $U = 15 \text{ m sec}^{-1}$  and  $\lambda \text{ (radar)} = \pi \text{ cm}$  were used in the calculation. The effect of varying the low wave number cutoff is small (on the order of a few decibels for wind speeds between 5 and  $20 \text{ m sec}^{-1}$ ) and is not illustrated. The effect of increasing wind speed is simply to cause a greater "tilting" of the small wave structure (which is primarily responsible for the scattering) by the larger waves. The result is a "smearing" of the incoherent scattered power pattern. Also not shown is the frequency dependence of the return. The frequency dependence can be expected to be small. This follows from the  $k$ -independence of both stationary phase and small perturbation approximations. (Note that  $k^3 S(2ka)$  is  $k$ -independent for the  $K^{-3}$  spectrum.)

A comparison of stationary phase and small perturbation approximations with the Kirchoff integral is made in Figure 4. A value of  $\sigma_x = 0.215$  was chosen to match  $\sigma_1^\circ$  with the Kirchoff  $\sigma^\circ$  at vertical incidence.

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\*The spectral constant is wind-speed dependent. The observations of Leykin and Rosenberg (1970), for example, show that  $A$  increases with wind speed until for high wind speeds  $A$  levels off and approaches its asymptotic (equilibrium) value.

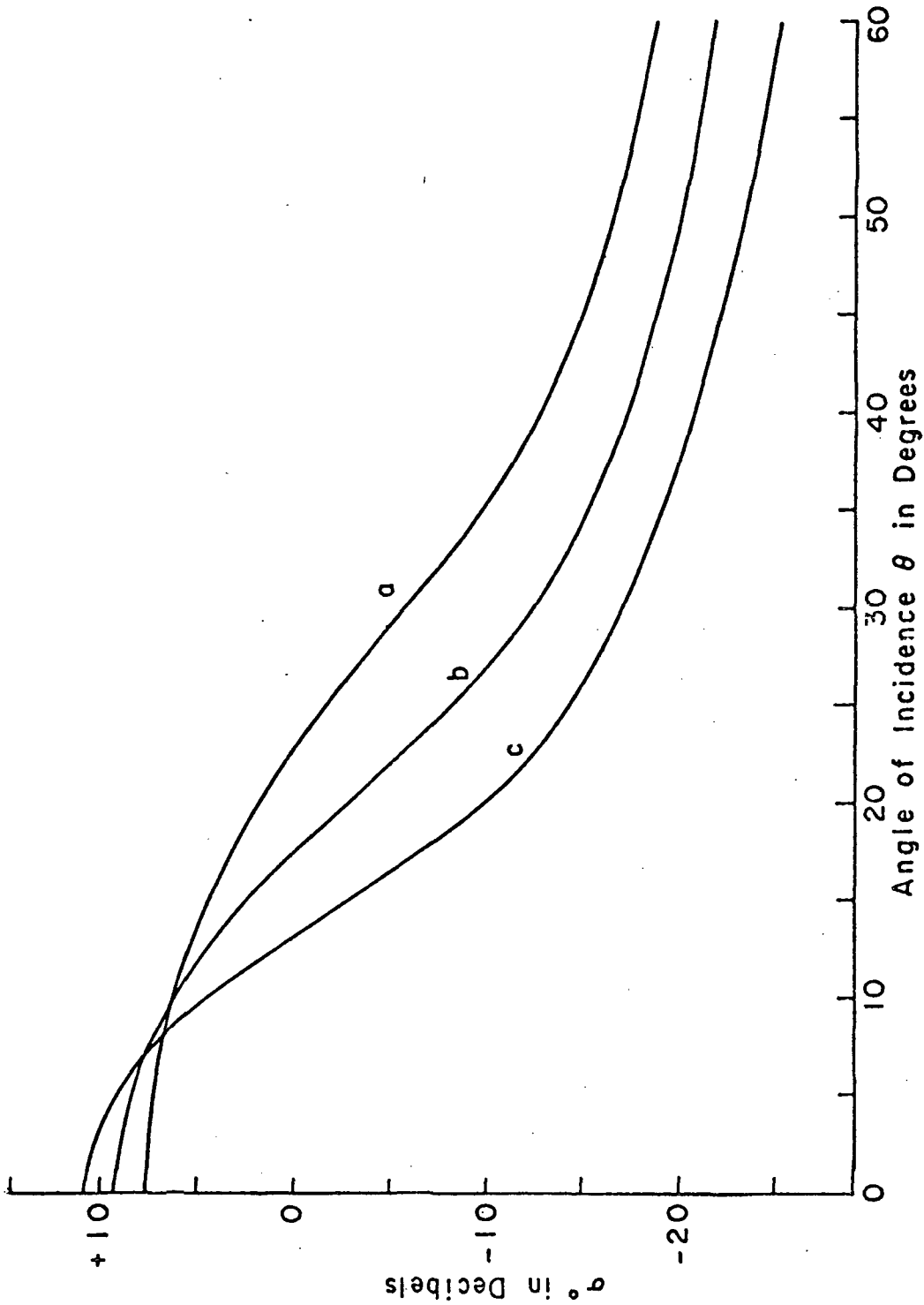


Fig. 3. Two dimensional Kirchhoff solution for a  $K^{-3}$  spectral-law surface. The curves 'a', 'b', and 'c' are respectively given by values of the spectral constant  $A = 5 \times 10^{-3}$ ,  $5/2 \times 10^{-3}$ , and  $5/4 \times 10^{-3}$ . Note that at large angles the cross section is proportional to  $A$ , and at vertical incidence is inversely proportional to the square root of  $A$ .

The coincidence of the small perturbation approximation ( $\sigma_2^\circ$ ) with the Kirchhoff curve provides a validation of small perturbation theory for sea-return from larger incidence angles.

The basic idea behind various "composite surface" models (e. g., Semyonov, 1966; Fung and Chan, 1969; Krishen, 1971) is illustrated in Fig. 5. The backscattered power from small perturbation and geometrical optics regimes is added incoherently,

$$\sigma^\circ = \sigma_1^\circ + \sigma_2^\circ, \quad (3.22)$$

where the specular (near vertical) portion of  $\sigma_2^\circ$  is suppressed. A remarkably close agreement with the full Kirchhoff integral is achieved.

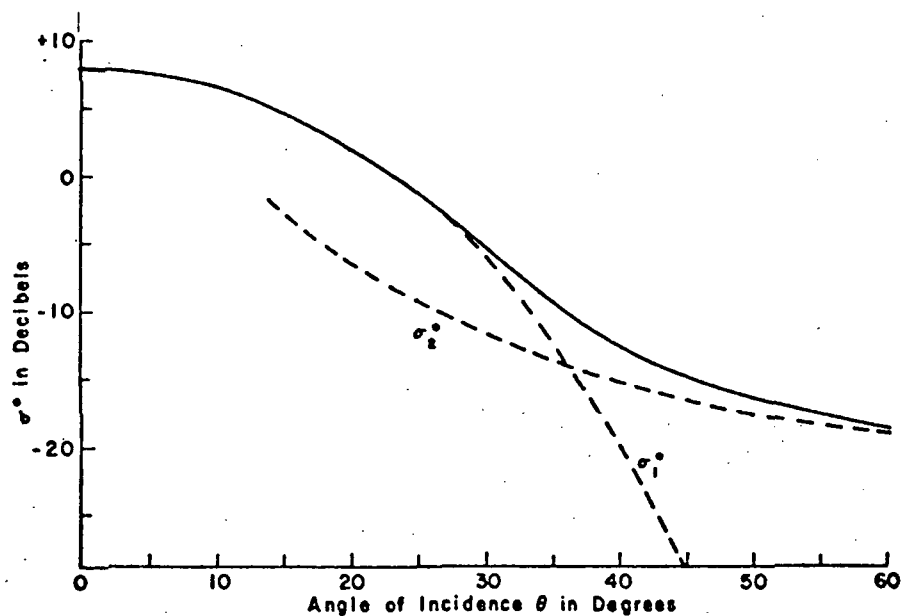


Fig. 4. Stationary phase and small perturbation approximations to the Kirchhoff integral. The Kirchhoff solution is the same as curve 'a' in Fig. 3.  $\sigma_1^0$  and  $\sigma_2^0$  are respectively given by eqs. (3.8) and (3.15).

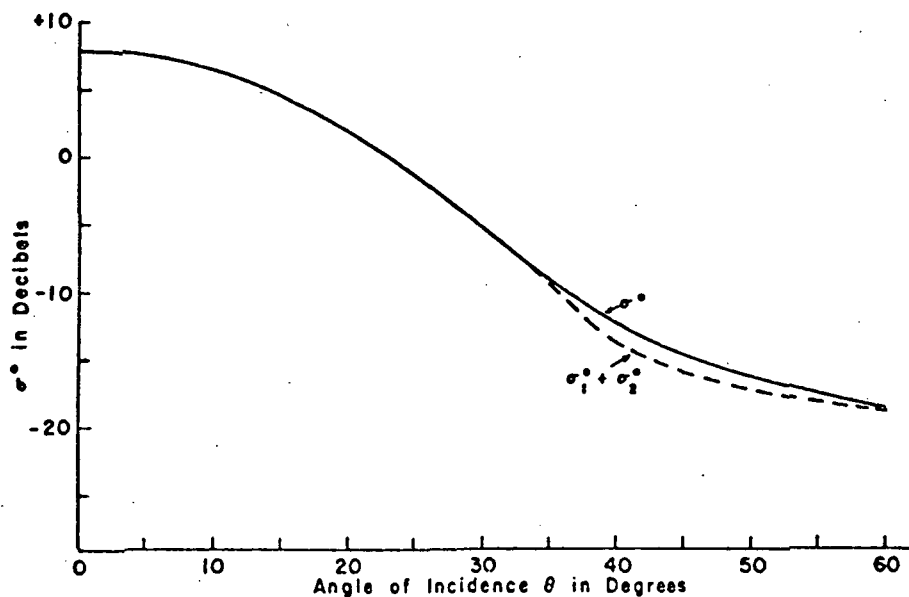


Fig. 5. The "composite surface" solution as a sum of stationary phase and small perturbation cross sections.

#### IV. Results of Small Perturbation Theory

Methods of small perturbation apply to surfaces for which

$$|f_x| \ll 1, \quad |kf| \ll 1. \quad (4.1)$$

The effect of surface roughness is then in the form of a small perturbation on the primary field on a perfectly flat horizontal surface. Various small perturbation methods have been advanced: for example, by Rice (1951), Bass (1961), and Wright (1966). Rice used a randomized Rayleigh method. The field and surface height are expanded in a Fourier series. The surface height is expressed as

$$f(x, y) = \sum_{m,n} P_{mn} e^{ia(mx+ny)} \quad (4.2)$$

where the  $P_{mn}$  are independent Gaussian random variables, and  $a = 2\pi/X$  is the fundamental wave number. First-order Rayleigh-Rice theory for perfect conductivity yields a far-zone cross section of the form

$$\sigma^\circ = 8\pi k^4 g(\theta) S(2ka, 0) \quad (4.3)$$

where the spectrum is defined by equation (3.12). The angle function  $g(\theta)$  depends on the polarization. For vertical polarization ( $E^i$ -vector in the  $x$ - $z$  plane),

$$g_{VV}(\theta) = (1 + \sin^2 \theta)^2. \quad (4.4a)$$

For horizontal polarization ( $E^i$ -vector parallel to  $y$ -axis);

$$g_{HH}(\theta) = \cos^4 \theta \quad (4.4b)$$

First-order Rayleigh-Rice theory gives no depolarization in the plane of incidence. To account for depolarization (cross-polarization), one



one must go to second order in the ordering parameters  $|kf|$  and  $|f_x|$ . This has been done by Valenzuela (1967). Fig. 6 compares the Kirchoff and Rayleigh-Rice solutions for the surface described by the  $K^{-3}$  spectrum.

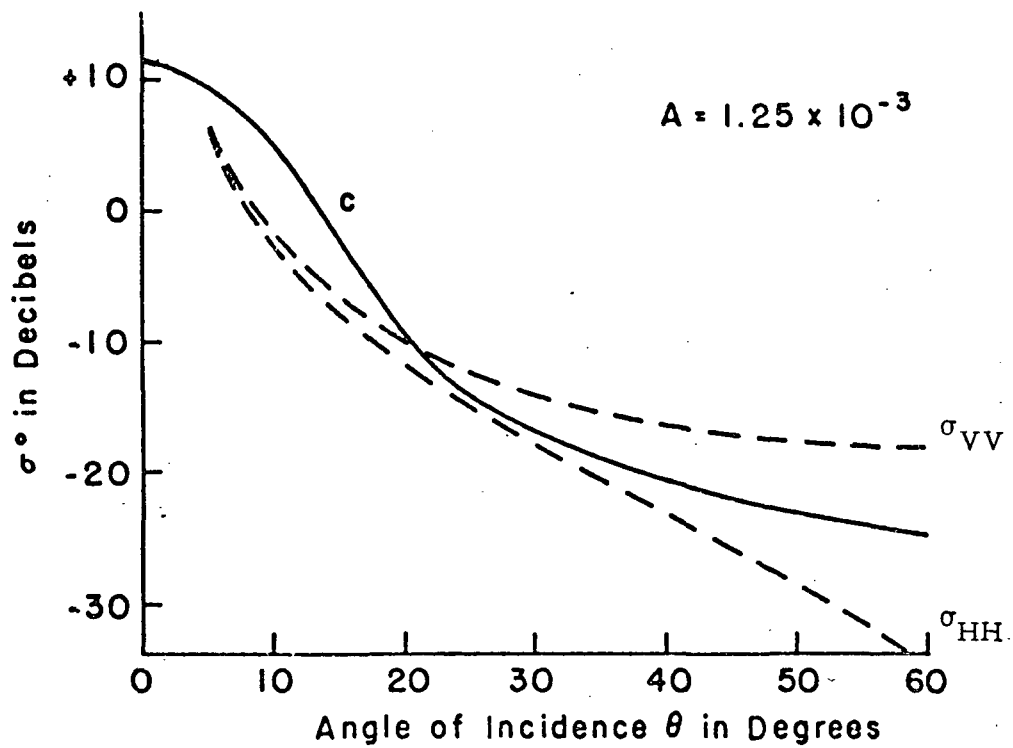
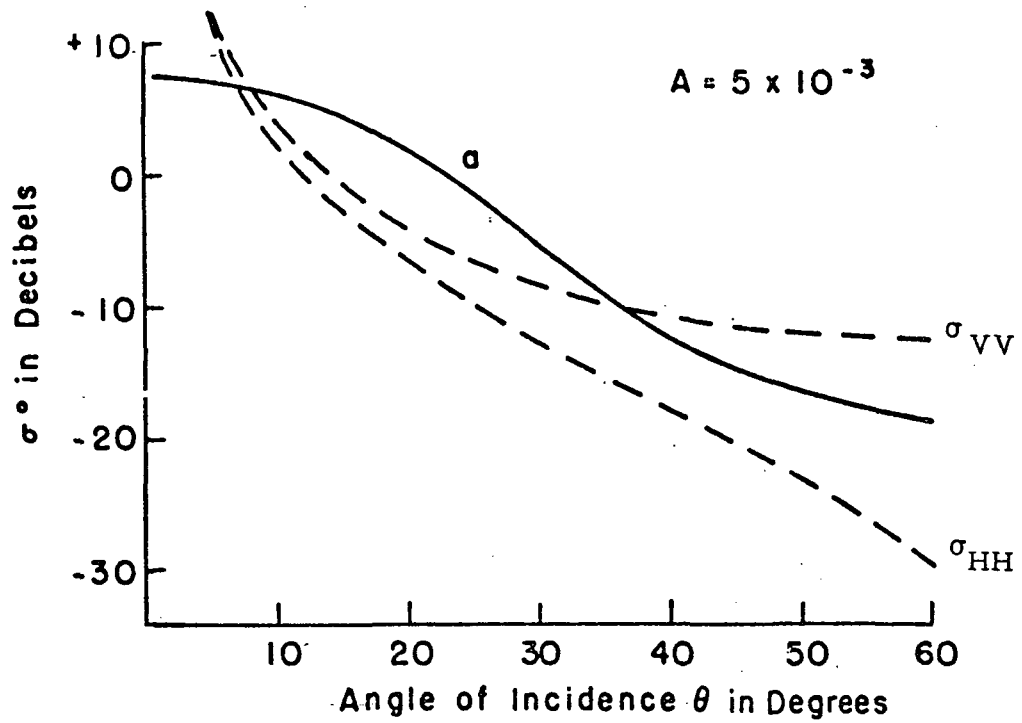


Fig. 6. First-order Rayleigh-Rice solution. The solid curves 'a' and 'c' are the same Kirchhoff curves shown in Fig. 3.

## V. A High-Frequency Correction to the Kirchoff Approximation

A simple correction to the Kirchoff approximation that accounts for the effect of surface curvature can be extracted from a field integral equation. For a perfectly conducting surface free from singular curves (cusps), the integral equation for the magnetic field is (e.g., Fock (1945)):

$$\underline{J}_s = \underline{J}_s^{(0)} + \hat{n} \times \frac{1}{2\pi} \int_{S_0} \underline{J}_s' \times \nabla' G \, dS' \quad (5.1)$$

where  $\underline{J}_s = \hat{n} \times \underline{H}$  is the surface current density and  $\underline{H}$  is the magnetic field on the surface; where:

$\underline{J}^{(0)} = \hat{n} \times 2\underline{H}^i$  is the Kirchoff value of the current density,

$\hat{n}$  is the unit surface normal directed outward from the conducting volume,

$\underline{H}^i$  is the incident magnetic field,

(') (prime) denotes source point coordinates  $\underline{x}'$  as opposed to field point coordinates,  $\underline{x}$ ,

$G = \exp(-ik|\underline{x} - \underline{x}'|)/|\underline{x} - \underline{x}'|$  is the Green's function for homogeneous space.

The assumption of a first-order continuous surface is consistent with high frequency approximation we shall be making. That is, as with the Kirchoff approximation, we shall require gentle curvature ( $\lambda/r_c \ll 1$ ). Mittra\* has derived a set of field integral equations which are valid for surfaces possessing sharp edges. These equations may provide a basis for future research. The

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\*Mittra, R.A.J.: Notes for a course given at the University of Illinois, Department of Electrical Engineering (unpublished).

formulation (5.1) is ideally suited to near-planar or smooth geometries. For, when the surface is locally nearly flat or nearly planar, the integral contribution is small compared to the Kirchoff value  $\hat{n} \times 2\mathbf{H}^i$  and can be regarded as a "small perturbation" on the Kirchoff value. This is seen from the fact that the vector  $\mathbf{J}_s' \times \nabla'G$  is oriented nearly parallel to the field point normal  $\hat{n}$ , so that the cross-product  $\hat{n} \times \mathbf{J}_s' \times \nabla'G$  is small.

Fock (1945) has examined the conditions under which a perfect conductivity formulation is valid. An assumption of perfect conductivity is strictly justifiable only when the radius of curvature is large compared to the skin depth (depth of penetration of the field). Conduction currents can then be considered to be confined to a thin skin layer, and can then be represented by a surface current density. Thus, we must have

$$\delta \ll r_c$$

where  $\delta$  is the skin depth (normal to a flat interface), the depth at which the field has attenuated by a factor of  $e^{-1}$ . For all microwave frequencies up to and including X-band (3 cm), sea water is a good conductor in the sense that the skin depth is much smaller than the wavelength. Even at X-band,  $\delta$  is small:  $\delta/\lambda \sim 1/2\pi$ .<sup>\*</sup> Thus, a perfect conductivity formulation of the integral equation is consistent with the high-frequency (small  $\lambda/r_c$ ) approximation we are making.

We develop the integral equation for the horizontal rough surface,  $z = f(x, y)$ . We use the following notation:

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<sup>\*</sup>From Saxton and Lane's (1952) data.

$$\underline{x} = (x, y, f), \quad \underline{x}' = (x', y', f')$$

$$\underline{\rho} = \underline{x}' - \underline{x} = (\xi, \eta, \zeta)$$

$$\hat{n} = (-f_x, -f_y, 1) \cos \omega$$

$$\cos \omega = (1 + f_x^2 + f_y^2)^{-\frac{1}{2}};$$

$$dS = \sec \omega \, dx \, dy$$

If we define the two-vector  $\underline{J}$  as

$$\underline{J} = \begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \hat{x} \cdot \underline{J}_s \\ \hat{y} \cdot \underline{J}_s \end{bmatrix} \sec \omega, \quad (5.2)$$

then the three-vector equation (5.1) can be written as the two-vector matrix equation,\*

$$\underline{J} = \underline{J}^{(0)} + \frac{1}{2\pi} \iint \psi \underline{\underline{M}} \underline{J}' \, d\xi \, d\eta \quad (5.3)$$

where

$$\psi = - (1 + ik\rho) \frac{e^{-ik\rho}}{\rho} \quad (5.4)$$

and  $\underline{\underline{M}} = \underline{\underline{M}}(\underline{x}; \underline{\xi})$  is the matrix

$$\underline{\underline{M}} = \begin{bmatrix} -\xi \frac{\partial \zeta}{\partial \xi} - \eta f_y + \zeta & -\xi \frac{\partial \zeta}{\partial y} \\ -\eta \frac{\partial \zeta}{\partial x} & -\eta \frac{\partial \zeta}{\partial \eta} - \xi f_x + \zeta \end{bmatrix} \quad (5.5)$$

In this notation  $\zeta$  is to be considered a function of  $\underline{x}$  and  $\underline{\xi}$ ,

$$\zeta = \zeta(\underline{x}; \underline{\xi})$$

so that

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\* See Jackson (1971), pp. 4-8, for details.

$$\frac{\partial \zeta}{\partial \xi} = f'_x \quad (x \rightarrow y)$$

and

$$\frac{\partial \zeta}{\partial x} = f'_x - f_x \quad (x \rightarrow y) .$$

The  $J_z$  component follows from the condition that  $\underline{J}_s$  is tangential to the surface:

$$J_z = f_x J_x + f_y J_y . \quad (5.6)$$

We relax the high frequency (small curvature) condition of the Kirchhoff approximation to allow for some degree of curvature. Since we are still dealing with a "smooth" surface, we can assume the surface height  $f$  has a Taylor series expansion about every point,  $\underline{x}_0$ :

$$f(\underline{x}) = f + f_x u + f_y v + f_{xx} \frac{u^2}{2} + f_{xy} uv + f_{yy} \frac{v^2}{2} + \dots .$$

From now on it will be understood that  $f$  and the derivatives are to be evaluated at the local origin  $\underline{x}_0$ . We shall have  $\underline{u} = (u, v)$  stand for the relative (horizontal) position vector of a field point. The relative position vector of a source point shall be given by

$\underline{x}' - \underline{x}_0 = \underline{u} + \underline{\xi}$ . If we form the difference  $\zeta = f' - f$  we get

$$\zeta = f_x \xi + f_y \eta + f_{xx} \left( u\xi + \frac{\xi^2}{2} \right) + f_{xy} (u\eta + v\xi + \xi\eta) + f_{yy} \left( v\eta + \frac{\eta^2}{2} \right) + \dots$$

A little algebra will show that the matrix  $\underline{\underline{M}}$  has the expansion

$$\underline{\underline{M}} = \underline{\underline{M}}(\underline{x}; \underline{\xi}) = \begin{bmatrix} -f_{xx} \frac{\xi^2}{2} + f_{yy} \frac{\eta^2}{2} & -f_{xy} \xi^2 - f_{yy} \xi \eta \\ -f_{xy} \eta^2 - f_{xx} \xi \eta & f_{xx} \frac{\xi^2}{2} - f_{yy} \frac{\eta^2}{2} \end{bmatrix} + \dots \quad (5.7)$$

Thus, to a first approximation,  $\underline{\underline{M}}$  is independent of the local field point coordinates  $\underline{u}$  and can be expressed in terms of the separation vector  $\underline{\xi}$  alone. Call this approximation  $\underline{\underline{M}}^{(2)}$ .

A first approximation to  $\psi$  is obtained by letting  $\rho$  correspond to the distance on the  $\underline{x}_0$  tangent plane,

$$\rho = \rho_1 = (a^2 \xi^2 + b^2 \eta^2)^{\frac{1}{2}} \quad (5.8)$$

where

$$a^2 = 1 + f_x^2, \quad b^2 = 1 + f_y^2. \quad (5.9)$$

If we let

$$\psi = \psi^{(1)} = - (1 + ik\rho_1) \frac{e^{-ik\rho_1}}{\rho_1} \quad (5.10)$$

the leading error term is proportional to the curvature.

If the approximation  $\psi \underline{\underline{M}} = \psi^{(1)} \underline{\underline{M}}^{(2)}$  is made the leading error term is proportional to third derivatives of surface height and the product of two second derivatives. What we are going to assume is that the bulk of the integral  $\iint \psi^{(1)} \underline{\underline{M}}^{(2)} \underline{J}' d\xi d\eta$  is formed in the neighborhood of  $\underline{x}_0$  where the error terms are small. This assumption is difficult to justify. Cullen (1958) has examined the high frequency behavior of the integral equation (5.1) for a convex body. There is an apparent contradiction in the relative importance of the source distribution near and removed from the field point. We will see this when we compare the radiative (far-source) contribution to the inductive (near-source) contribution to the integral (see below, p. 39). We should expect the mild curvature restriction on this approximation to be very similar to the criteria (inequalities (2.7))

for the Kirchoff approximation, except that this approximation should be good to  $O(\lambda^2/r_c^2)$  rather than  $O(\lambda/r_c)$ .

In any event, assuming  $\psi \underline{\underline{M}} = \psi^{(1)} \underline{\underline{M}}^{(2)}$ , the integral equation (5.3) becomes

$$\underline{\underline{J}}(\underline{u}) = \underline{\underline{J}}^{(o)}(\underline{u}) + \frac{1}{2\pi} \iint \psi^{(1)}(\rho_1) \underline{\underline{M}}^{(2)}(\underline{\xi}) \underline{\underline{J}}(\underline{u} + \underline{\xi}) d\underline{\xi} d\eta \quad (5.11)$$

Equation (5.11) can be solved exactly by Fourier transformation techniques. Fourier transformation is now a common method for solving two-dimensional (plane) diffraction problems, where integral equations of this type occur (Bouwkamp, 1954). But, remember, unlike a true two-dimensional equation, eq. (5.11) is only approximately correct. The older method of iterated kernels is a more appropriate method of solution.

The integral operation takes an  $O(1)$  quantity into an  $O(\lambda/r_c)$  quantity; and  $O(\lambda/r_c)$  quantity into an  $O(\lambda^2/r_c^2)$  quantity, and so on. Since  $\underline{\underline{J}}$  differs from  $\underline{\underline{J}}^{(o)}$  by an  $O(\lambda/r_c)$  quantity, we can set

$$\underline{\underline{J}} = \underline{\underline{J}}^{(o)} + \underline{\underline{J}}^{(1)} \quad (5.12)$$

where  $\underline{\underline{J}}^{(o)} = O(1)$  (for a unit magnetic field) and  $\underline{\underline{J}}^{(1)} = O(\lambda/r_c)$ .

Equation (5.11) then yields for  $\underline{\underline{J}}^{(1)}$ :

$$\underline{\underline{J}}^{(1)}(\underline{u}) = \frac{1}{2\pi} \iint \psi^{(1)}(\rho_1) \underline{\underline{M}}^{(2)}(\underline{\xi}) \underline{\underline{J}}^{(o)}(\underline{u} + \underline{\xi}) d\underline{\xi} d\eta + O(\lambda^2/r_c^2).$$

We have really lost nothing here since equation (5.11) was accurate only to  $O(\lambda/r_c)$  to start with. Note that the above equation is most accurate at the local origin  $\underline{u}$ . And since we no longer need



the convolution properties of the original equation because  $\underline{J}^{(0)}$  is a known function, all we need calculate is

$$\underline{J}^{(1)} = \frac{1}{2\pi} \iint \psi^{(1)} \underline{M}^{(2)} \underline{J}^{(0)} d\xi d\eta \quad (5.13)$$

For a plane-wave incident field with unit magnetic vector,  $\underline{J}^{(0)}$  has the form

$$\underline{J}^{(0)} = 2\underline{A} e^{-ik(ax-\gamma f)} \quad (5.14)$$

where for vertical polarization ( $\underline{E}^i$ -vector in the x-z plane):

$$\underline{A}^V = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (5.15a)$$

and for horizontal polarization ( $\underline{E}^i$ -vector parallel to the y-axis):

$$\underline{A}^H = \begin{bmatrix} -af_y \\ \gamma + af_x \end{bmatrix} \quad (5.15b)$$

It is entirely consistent with the development of equation (5.13) to expand  $\underline{J}^{(0)}$  in terms of  $\underline{\xi}$  about  $\underline{x}_0$  neglecting terms of  $O(\lambda/r_c)$ . We can do this because when multiplied by the kernel,  $O(\lambda/r_c)$  terms become  $O(\lambda^2/r_c^2)$  terms. We then have approximately

$$\underline{J}^{(0)} = 2\underline{A} e^{-ik(ax-\gamma f)} \cdot e^{-i\underline{\ell} \cdot \underline{\xi}} \quad (5.16)$$

where  $\underline{A}$  is equal to (5.15) with  $f_x$  and  $f_y$  evaluated at the local origin,  $\underline{x}$ . (The sub-zero notation is abandoned.) And  $\underline{\ell}$  is the wave number,

$$\underline{\ell} = k(a - \gamma f_x, -\gamma f_y) \quad (5.17)$$

Putting  $\underline{J}^{(0)}$  in the integral (5.13) we get

$$\underline{J}^{(1)} = \underline{S} \underline{J}^{(0)} \quad (5.18)$$

where  $\underline{\underline{S}}$  is the Fourier integral,

$$\underline{\underline{S}} = \frac{1}{2\pi} \iint_{-\infty}^{\infty} \psi^{(1)} \underline{\underline{M}}^{(2)} e^{-i\ell \cdot \underline{\xi}} d\xi d\eta \quad (5.19)$$

The infinite limits have been applied just to make the integral definite.

The evaluation of this integral is given in the Appendix.

The elements  $S_{ij}$  of  $\underline{\underline{S}}$  are found to be:

$$\begin{aligned} S_{11} &= \frac{-i}{2kab\nu^3} \left[ (1 - m^2) \frac{f_{xx}}{a^2} - (1 - \ell^2) \frac{f_{yy}}{b^2} \right] \\ S_{12} &= \frac{-i}{kab\nu^3} \left[ (1 - m^2) \frac{f_{xy}}{a^2} + \ell m \frac{f_{yy}}{ab} \right] \\ S_{21} &= \frac{-i}{kab\nu^3} \left[ (1 - \ell^2) \frac{f_{xy}}{b^2} + \ell m \frac{f_{xx}}{ab} \right] \\ S_{22} &= -S_{11} \end{aligned} \quad (5.20)$$

where we have defined

$$\ell = (a - \gamma f_x)/a, \quad m = -\gamma f_y/b \quad (5.21)$$

and

$$\nu = (1 - \ell^2 - m^2)^{1/2} \quad (5.22)$$

and where again we have

$$a^2 = 1 + f_x^2, \quad b^2 = 1 + f_y^2.$$

The wave numbers  $k\ell$  and  $km$  are the projections of the propagation vector  $\underline{k}_1$  onto the tangent plane in the  $x$  and  $y$  directions respectively. The quantity  $\nu$  is a bit difficult to interpret, but it can be written in terms of the "tilt angles" of the surface, defined by  $\tan \psi = f_x$  and  $\tan \delta = f_y$ :

$$\nu^2 = \cos^2(\theta - \psi) - \cos^2\theta \sin^2\delta \quad (5.23)$$

At (locally) normal incidence,  $\psi = \theta$  and  $\delta = 0$  so that  $\nu = 1$  (its maximum value). For local incidence near grazing ( $\psi = \theta - \pi/2$ )  $\nu$  may go to zero causing  $\underline{S}$  to blow up. Since  $\underline{S}$  must be small if it is to be a good approximation, large incidence angles must be avoided.

The correction  $S_{11}$  is seen to be identical to an asymptotic form found by Fock (1945) for the current distribution on a convex paraboloid of revolution. Fock did not use the integral equation, but rather solved Maxwell's equations by separation of variables. Fock's asymptotic form holds for large distances away from the shadow boundary.

Jackson (1971) has shown that the corrected current density  $\underline{J}^{(0)} + \underline{J}^{(1)}$  corresponds with Rice's current density in the case of gentle curvature and moderate incidence angles.

It is interesting to compare the criteria (inequalities (2.7)) with our correction. For the one-dimensional case,  $f = f(x)$ , alone, say, the condition that  $\underline{S}$  be small gives

$$\frac{1}{2k\nu^3} \cdot \frac{|f_{xx}|}{a^3} \ll 1.$$

Or, since  $|f_{xx}|/a^3 = r_c^{-1}$ ,  $\nu = \cos(\theta - \psi) = \cos\theta'$ , this means that

$$4\pi r_c \cos^3\theta' \gg \lambda \quad (5.24)$$

Note that  $\underline{S}$  is purely imaginary in number. This means that the perturbed current density  $\underline{J}^{(1)}$  is  $90^\circ$  out of phase with the zeroth-order current. Exactly how this phase shift will

determine the scattered field will depend on the height-curvature correlation properties of the surface.

Returning to the point made earlier, that there is an apparent contradiction in the integral equation between the local nature of the high frequency approximation and the importance of far-source contributions, we examine the integral  $\underline{S}$  (see the Appendix). The inductive contribution comes from the integrals

$$\text{Ind.} = \int_0^{\infty} J_n(L\rho_1) e^{-ik\rho_1} d\rho_1, \quad n = 0, 2$$

and the radiative contribution comes from the integrals,

$$\text{Rad.} = \int_0^{\infty} ik\rho_1 J_n(L\rho_1) e^{-ik\rho_1} d\rho_1, \quad n = 0, 2.$$

The ratio  $\text{Ind}/\text{Rad} = \text{Rad} = \nu^{-2}$  for  $n = 0$  and  $2\nu^{-1} + \nu^{-2}$  for  $n = 2$ .

Thus, inductive and radiative contributions are comparable: this despite an intuition that the inductive component might preponderate. The rapid increase of the ratio with incidence angle ( $\nu \propto \sec^3 \theta$ ) warns us that far-source contributions are becoming more important, and that the surface curvature must be increasingly mild if the local nature of the solution is not to be violated.

To summarize our results, we write the corrected current density  $\underline{J}$  as

$$\underline{J} = \underline{J}^{(0)} + \underline{J}^{(1)} = (\underline{I} + \underline{S}) \underline{J}^{(0)} \quad (5.25)$$

where  $\underline{I}$  is the identity matrix.

## VI. Calculation of the Scattered Power

We apply our results to the calculation of the average power backscattered from a random rough surface. In the case of perfect conductivity, the far-field Stratton-Chu integral\* reduces to:

$$\underline{E}(\underline{R}) = \frac{-ik}{4\pi R} e^{-ikR} \hat{R} \times \iint_{A_0} \hat{R} \times (-\eta \underline{J}_s \sec \omega) e^{ik\hat{R} \cdot \underline{x}} dxdy. \quad (6.1)$$

$\underline{E}$  is the electric field vector,  $\underline{R} = R \hat{R}$  is the position vector of the far field (Fraunhofer zone) point;  $\underline{x}$  is the position vector of the source points on the surface;  $A_0$  is the illuminated area; and  $\eta$  is the impedance of free space.

For backscatter, the unit vector  $\hat{R}$  is directed toward the source of incident radiation and so is given by

$$\hat{R} = (-\alpha, 0, \gamma).$$

Since in the far field the E-vector oscillates transversely to the propagation vector  $k\hat{R}$ , we have

$$\underline{E} \cdot \hat{R} = 0;$$

hence only two components are needed to specify  $\underline{E}$ . In practice the "horizontal" and "vertical" components are used,

$$E^H = E_y$$

and

$$E^V = -\gamma^{-1} E_x.$$

For a unit incident electric field, we have

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\*For example, see Silver (1949), p. 161. The Stratton-Chu integral is a particular vector form of the Helmholtz integral, eq. (2.1).

$$\eta \underline{J}_s \sec \omega = \begin{bmatrix} J_x \\ J_y \\ f_x J_x + f_y J_y \end{bmatrix};$$

and  $J_x$  and  $J_y$  are given by equation (5.25). (We are now assuming a unit incident electric vector.) Expressing the amplitude vector of the incident field  $\underline{A}$  explicitly for H and V polarizations, but keeping the symbols  $S_{ij}$  for the elements of  $\underline{S}$ , equation (6.1) yields for the four polarization combinations:

$$E^{HH} = C \iint [(\gamma + a f_x)(1 - S_{11}) - a f_y S_{21}] e^{-i2k(ax - \gamma f)} dx dy \quad (6.2a)$$

$$E^{HV} = -C \iint [(\gamma + a f_x)^2 S_{12} - 2a f_y (\gamma + a f_y) S_{11} - (a f_y)^2 S_{21}] e^{-i2k(ax - \gamma f)} dx dy \quad (6.2b)$$

$$E^{VH} = C \iint S_{21} e^{-i2k(ax - \gamma f)} dx dy \quad (6.2c)$$

$$E^{VV} = -C \iint [(\gamma + a f_x)(1 + S_{11}) + a f_y S_{21}] e^{-i2k(ax - \gamma f)} dx dy \quad (6.2d)$$

The first H or V stands for a horizontally or vertically polarized incident wave; the second H or V stands for the horizontal or vertical component of the backscattered wave. In the above, we have used the fact that  $S_{22} = -S_{11}$ ; we have let C stand for  $-ik \exp(-ikR)/2\pi R$ .

The scattered power is proportional to  $|E|^2$ . The usual way to calculate  $|E|^2$  is to form the two-fold integral from  $|E| = E E^*$ . The integrals (6.2) are of the form

$$E = C \iint (K + P) e^{-i2k(ax - \gamma f)} dx dy \quad (6.3)$$

where  $K$  is the Kirchhoff term, equal to  $\gamma + \alpha f_y$  for co-polarization (HH, VV) and equal to zero for cross-polarization (HV, VH).  $P$  is the perturbation part. The forms (6.3) yield for the co-polarized  $EE^*$ :

$$EE^* = CC^* \iiint (KK' + K'P^* + KP' + P'P^*) e^{-i2k[\alpha(x'-x) - \gamma(f'-f)]} dx dy dx' dy'; \quad (6.4a)$$

and for the cross-polarized  $EE^*$ :

$$EE^* = CC^* \iiint P'P^* e^{-i2k[\alpha(x'-x) - \gamma(f'-f)]} dx dy dx' dy' \quad (6.4b)$$

Jackson (1971) discarded the  $P'P^*$  terms in co-polarized returns on the grounds that their magnitude was of the same order as the error in the cross-product terms, i.e., of  $O(\lambda^2/r_c^2)$ . This is a reasonable choice for small angles of incidence where the  $K$  terms are of  $O(1)$ . Toward high angles of incidence, however,  $\gamma + \alpha f_x$  becomes appreciably less than unity, and can have a magnitude  $O(f_x) = O(\lambda/r_c)$ . Thus, at large incidence angles the error in the cross-product terms may be significantly less than the magnitude of the  $P'P^*$  terms, justifying the retention of the  $P'P^*$  terms. Also, we must consider the fact that we do not know how the errors will transform through the integral operation. In any event, we should not want to lose possibly valid information, and moreover, we should like a guarantee of the power being a positive quantity by keeping a completed square.

We consider a rough surface  $z = f(x, y)$  to be a realization of a stationary (homogeneous) random process. The return powers are then random variables. For an illuminated area  $A_0$  very much larger

than the scale of roughness, one might expect the variability in return power to be small. However, this ignores the fact that scattered radiation from different portions of  $A_0$  will have random relative phases, resulting with Rayleigh-type statistics of signal fading and reinforcement. To calculate the average power we proceed by taking an ensemble average or expectation of all possible surface realizations, denoted by corner brackets  $\langle \dots \rangle$ . Since expectation and integral operations are commutative, the average power return in (6.4a) can be written as

$$\langle |E|^2 \rangle = CC^* \iiint \langle K'K + K'P^* + KP' + P'P^* \rangle \cdot e^{i2k\gamma(f' - f)} e^{-i2ka(x' - x)} dx dy dx' dy' ,$$

and similarly for (6.4b).

An immediate consequence of stationarity is that expectations of the type

$$\langle K'P^* + KP' \rangle e^{i2k\gamma(f' - f)}$$

can be expressed as

$$\Phi(\underline{\xi}) + \Phi^*(-\underline{\xi})$$

where

$$\Phi = \langle K'P^* e^{i2k\gamma(f' - f)} \rangle$$

and

$$\underline{\xi} = \underline{x}' - \underline{x} .$$

The expectations are computed on the assumption that  $f$  is a stationary Gaussian random process of zero mean,  $\langle f \rangle = 0$ . Define the twelve dimensional random vector  $\underline{Y}$  whose first six elements are  $f, f_x, f_y, f_{xx}, f_{xy}, f_{yy}$  and whose second six elements are  $f', f'_x, f'_y, f'_{xx}, f'_{xy}, f'_{yy}$ , etc. The mean of any derivative of a stationary process is zero; hence



$$\langle \underline{Y} \rangle = \underline{0}.$$

Since the mean of the vector is zero the covariance matrix  $\underline{\underline{\Lambda}}$  can be written as

$$\underline{\underline{\Lambda}}(\underline{\xi}) = (\lambda_{ij}) = \langle Y_i Y_j \rangle \quad (6.6)$$

The multivariate Gaussian distribution with the covariance  $\underline{\underline{\Lambda}}$  has the probability density function,

$$p(\underline{y}) = \frac{1}{(2\pi)^6 (\det \underline{\underline{\Lambda}})^{\frac{1}{2}}} \exp \left\{ -\frac{1}{2} \underline{y}^T \underline{\underline{\Lambda}}^{-1} \underline{y} \right\} \quad (6.7)$$

where  $\underline{y}^T$  is the transpose of  $\underline{y}$  and  $\underline{\underline{\Lambda}}^{-1}$  is the inverse of  $\underline{\underline{\Lambda}}$ . Define the characteristic function of  $\underline{Y}$ ,

$$\phi(\underline{t}) = \langle e^{i \underline{t} \cdot \underline{Y}} \rangle; \quad (6.8a)$$

$$\phi(\underline{t}) = \int_{-\infty}^{\infty} dy_1 \int_{-\infty}^{\infty} dy_2 \cdots \int_{-\infty}^{\infty} dy_{12} e^{i \underline{t} \cdot \underline{Y}} p(\underline{y}). \quad (6.8b)$$

If  $p(\underline{y})$  is the multivariate normal distribution (6.7), then  $\phi$  has the form\*:

$$\phi(\underline{t}) = \exp \left\{ -\frac{1}{2} \underline{t}^T \underline{\underline{\Lambda}} \underline{t} \right\}.$$

Or, in terms of elements  $t_i$ ,

$$\phi(t_1, t_2, \dots, t_{12}) = \exp \left\{ -\frac{1}{2} \sum \lambda_{ij} t_i t_j \right\} \quad (6.9)$$

Now, the required expectations can be generated in a simple manner from the characteristic function by expressing the slope-dependent coefficients in  $\underline{\underline{S}}$  as polynomials in  $f_x$  and  $f_y$ . We expand the (three) slope-coefficients in (5.20) in a Taylor series about

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\*E.g., Wilks (1962), p. 168.

$f_x = f_y \approx 0$ . (Expansion about the rms values  $\sqrt{\lambda_{22}}$ ,  $\sqrt{\lambda_{33}}$  might be more sensible, but it is a good bit more difficult.) We can truncate at first, second, or third order in slope. With the multipliers of the phasor  $\exp\{i2k_Y(f' - f)\}$  expressed as polynomials in the  $Y$ -elements we can compute term by term the expectations of the forms:

$$\begin{aligned}
 & \langle e^{i2k_Y(Y_7 - Y_1)} \rangle \\
 & \langle Y_p e^{i2k_Y(Y_7 - Y_1)} \rangle \\
 & \langle Y_p Y_q e^{i2k_Y(Y_7 - Y_1)} \rangle \\
 & \vdots
 \end{aligned} \tag{6.10}$$

These averages are computed from the characteristic function in the manner outlined: Define the twelve dimensional vector  $\underline{t}^*$  (star does not mean complex conjugate) all of whose elements are zero except for  $t_1^*$  and  $t_7^*$  which have the values,

$$\begin{aligned}
 t_1^* &= -2k_Y \\
 t_7^* &= +2k_Y
 \end{aligned} \tag{6.11}$$

The expectations (6.10) can then be written as

$$\begin{aligned}
 & \langle e^{i\underline{t}^* \cdot \underline{Y}} \rangle \\
 & \langle Y_p e^{i\underline{t}^* \cdot \underline{Y}} \rangle \\
 & \langle Y_p Y_q e^{i\underline{t}^* \cdot \underline{Y}} \rangle \\
 & \vdots
 \end{aligned}$$

From the definition of the characteristic function (6.8), we find

$$\begin{aligned}
\langle e^{i \underline{t}^* \cdot \underline{Y}} \rangle &= \phi(\underline{t}^*) \\
\langle Y_p e^{i \underline{t}^* \cdot \underline{Y}} \rangle &= i^{-1} \left. \frac{\partial \phi}{\partial t_p} \right|_{\underline{t} = \underline{t}^*} \\
\langle Y_p Y_q e^{i \underline{t}^* \cdot \underline{Y}} \rangle &= i^{-2} \left. \frac{\partial^2 \phi}{\partial t_p \partial t_q} \right|_{\underline{t} = \underline{t}^*} \\
&\vdots
\end{aligned} \tag{6.12}$$

From (6.9) and the definition of  $\underline{t}^*$  (6.11) we find

$$\begin{aligned}
\phi(\underline{t}^*) &= \exp\{-4k^2 \gamma^2 (\lambda_{11} - \lambda_{17})\} \\
\left. \frac{\partial \phi}{\partial t_p} \right|_{\underline{t}^*} &= 2k\gamma (\lambda_{p1} - \lambda_{p7}) \phi(\underline{t}^*) \\
\left. \frac{\partial^2 \phi}{\partial t_p \partial t_q} \right|_{\underline{t}^*} &= [4k^2 \gamma^2 (-\lambda_{q1} + \lambda_{q7})(-\lambda_{p1} + \lambda_{p7}) - \lambda_{pq}] \phi(\underline{t}^*) \\
&\vdots
\end{aligned} \tag{6.13}$$

In the manner outlined, the required expectations can be calculated. The last step is to find the covariances  $\lambda_{ij}$  as a function of the lag  $\underline{\xi} = \underline{x}' - \underline{x}$ . All 72 covariances can be expressed as partial derivatives of the covariance function,

$$R(\underline{\xi}) = \langle f'f \rangle.$$

In accordance with our "smoothness" condition,  $R(\underline{\xi})$  possesses continuous partial derivatives of all orders.

If the illuminated area  $A_0$  is large compared to the scales of roughness in the  $x$  and  $y$  directions ("correlation lengths"), then the scattering integrals of the form

$$\left(\frac{k}{2\pi R}\right)^2 \iiint C(\xi) e^{-i2ka(x'-x)} dx dy dx' dy' ,$$

are nearly equal to

$$\left(\frac{k}{2\pi R}\right)^2 A_0 \int_{-\infty}^{\infty} \int_{-Y}^Y C(\xi) e^{-i2ka\xi} d\xi d\eta .$$

$C(\xi)$  stands for the expectation  $\langle \{ \dots \} e^{i2k\eta(f'-f)} \rangle$  and  $Y$  is a large distance in the  $\eta$  direction. Because of the behavior of the exponential  $\phi$ ,  $Y$  can go to infinity only in the sense of the limit

$$\lim_{Y \rightarrow \infty} \int_{-\infty}^{\infty} \int_{-Y}^Y C(\xi) e^{-i2ka\xi} d\xi d\eta ;$$

and it is in this limiting sense that the infinite limits of integration in the final formulas have been applied.\*

The power returns are usually given in terms of the normalized isotropic radar backscatter cross sections,  $\sigma^\circ$ , defined by

$$\sigma^\circ = \frac{4\pi R^2}{A_0} \langle |E|^2 \rangle .$$

$\langle |E|^2 \rangle$  is the quantity we have calculated, namely, the ratio of  $\langle |E|^2 \rangle$  at the receiver to  $|E|^2$  incident. Here, we do not consider realistic antenna gain patterns. The incident field is taken to be of constant amplitude over the area  $A_0$ .\*\*

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\* Cf. pp. 13, 14.

\*\*Requiring the incident field to be of constant amplitude is only an artificial restriction. The cross-section is really determined by the statistical structure of a small patch, over which the antenna gain pattern is practically constant. The effect of gain pattern can be accounted for formally by introducing an effective area  $A_e$  in place of  $A_0$ .

Following are the scattering integrals arrived at. The co-polarized returns are expressed as

$$\begin{aligned}\sigma_{HH}^{\circ} &= K + P + S \\ \sigma_{VV}^{\circ} &= K - P + S\end{aligned}\tag{6.14}$$

where K represents the "Kirchoff" part, P the "perturbation" part, and S represents the perturbation "squared". The P-term is calculated to second order in slope, and the S-term is calculated to zeroth order in slope. To first order in slope, the cross-polarized returns are equal:

$$\sigma_{HV}^{\circ} = \sigma_{VH}^{\circ} = \sigma_D^{\circ}\tag{6.15}$$

where D stands for "depolarized" return. As previously defined, B stands for

$$B = -R(\underline{0}) + R(\underline{\xi}) ;$$

we introduce the additional symbols:

$$B_{\xi\xi} = -R_{\xi\xi}(\underline{0}) + R_{\xi\xi}(\underline{\xi}) \quad , \text{ etc.}$$

We find, then, that:

$$K = \frac{2k^2}{\pi} \int_0^\infty d\xi \int_{-\infty}^\infty d\eta \left\{ [\gamma^2 - a^2(R_{\xi\xi}^2 + 4k^2\gamma^2 R_\xi^2)] \cos 2ka\xi \right. \\ \left. - ik\gamma^2 a R_\xi \sin 2ka\xi \right\} e^{4k^2\gamma^2 B} \quad (16a)$$

$$P = \frac{2k^2}{\pi} \int_0^\infty d\xi \int_{-\infty}^\infty d\eta \left\{ [2(B_{\xi\xi} - \gamma^2 B_{\eta\eta}) - 8k^2 a^2 B_{\xi\xi}(R_\xi^2 - \gamma^2 R_\eta^2) \right. \\ + 4a^2\gamma^2 B_{\xi\xi}^2 - 2a^2\gamma^2 B_{\xi\xi}(R_{\xi\xi}(0) - \gamma^2 R_{\eta\eta}(0)) \\ + 4a^2\gamma^2 (R_\xi R_{\xi\xi\xi} - \gamma^2 R_\eta R_{\xi\xi\eta})] \cos 2ka\xi \\ + [4ka(R_\xi B_{\xi\xi} - 2\gamma^2 R_\eta B_{\xi\eta} + \gamma^2 R_\xi B_{\eta\eta}) \\ \left. - k^{-1} a \gamma^2 (R_{\xi\xi\xi} - \gamma^2 R_{\xi\eta\eta})] \sin 2ka \right\} e^{4k^2\gamma^2 B} \quad (16b)$$

$$S = \frac{2k^2}{\pi} \int_0^\infty d\xi \int_{-\infty}^\infty d\eta \left\{ \gamma^{-2} (B_{\xi\xi} - \gamma^2 B_{\eta\eta})^2 \right. \\ \left. + \frac{k^{-2}\gamma^{-4}}{4} (R_{\xi\xi\xi\xi} - 2\gamma^2 R_{\xi\xi\eta\eta} + \gamma^4 R_{\eta\eta\eta\eta}) \right\} \cdot \cos 2ka\xi e^{4k^2\gamma^2 B} \quad (16c)$$

$$\sigma_D^\circ = \frac{2k^2}{\pi} \int_0^\infty d\xi \int_{-\infty}^\infty d\eta \left\{ [4B_{\xi\eta}^2 + (k\gamma)^{-2} R_{\xi\xi\eta\eta}] \cos 2ka\xi \right. \\ + [16kaB_{\xi\eta}(R_\xi B_{\xi\eta} + R_\eta B_{\xi\xi}) \\ \left. + 4k^{-1}\gamma^{-2} a (B_{\xi\xi} R_{\xi\eta\eta} + B_{\xi\eta} R_{\xi\xi\eta} + R_\eta R_{\xi\xi\xi\eta} + R_\xi R_{\xi\xi\eta\eta})] \sin 2ka\xi \right\} e^{4k^2\gamma^2 B} \quad (16d)$$

The Kirchoff term is in the form used by Chia (1968). Beckmann's form, eq. (2.22), differs because of the integration by parts and the neglect of edge terms. The K-term can be integrated by parts, yielding Beckmann's form upon neglecting "edge terms". For most practical applications, we have deep fade conditions,  $2k\gamma\sigma \gg 1$ , and a large  $A_0$ . One need only be careful of edge effects when numerically computing the integral for large angles approaching grazing incidence (this would entail having to carry the integration over a larger lag).

## VII. Properties of the Solution

Here we examine the general nature of the solution, give a sample calculation, and present for comparison some published sea-return data.

### 7.1 Comparison with the results of first-order Rayleigh-Rice theory

It was shown in section 3.3 that for values of the spectral constant  $A$  typical of the sea, the small perturbation approximation becomes very accurate toward large incidence angles. This fact demonstrates the validity of small-perturbation methods for sea-return from angles of incidence exceeding  $\theta = 45^\circ$  or so. We should like to compare our solution with Rayleigh-Rice theory for the condition of small roughness amplitude (i. e., for  $2k_Y\sigma \ll 1$ ). If we linearize the integrands of the scattering integrals (6.16) and integrate using the definition of the spectrum (eq. (3.12)), we get scattering cross-sections of the form:

$$\sigma^\circ = 8\pi k^4 g(\theta) S(2ka, 0)$$

where

$$g_{HH}(\theta) = (1 - \tan^2 \theta)^2 \quad (7.1a)$$

$$g_{VV}(\theta) = (1 + \tan^2 \theta)^2 \quad (7.1b)$$

and the depolarized return is zero. Comparing this with Rice's  $g(\theta)$ , eqs. (4.4), viz.,

$$g_{HH}(\theta) = \cos^4 \theta = (1 - \sin^2 \theta)^2$$

$$g_{VV}(\theta) = (1 + \sin^2 \theta)^2$$

we see that (7.1) is in good agreement for  $\theta \lesssim 30^\circ$ . The failure to



match Rice's  $g(\theta)$  for large incidence angles indicates a general failure of our solution for representing backscatter from any surface having an appreciable spectral density at the first-order Bragg condition ( $K = 2ka$ ) for  $\theta \gtrsim 40^\circ$ . Clearly, the simple curvature correction is over-predicting the splitting of the copolarized returns from these angles. The failure of representation at large angles is not surprising, and could have been expected on the basis of the criterion (5.24).

## 7.2 A sample calculation: Copolarized returns from an isotropic $K^{-4}$ spectral-law surface and a one-dimensional $K^{-3}$ spectral-law surface

We should like to apply our formulas to scattering from an isotropic surface described by the simple  $K^{-4}$  spectral law (eq. (3.17)). It is understood that a  $K^{-4}$  law surface is extremely rough (having first-order discontinuities) and poorly conforms with our requirements of smoothness. However, as the  $K^{-4}$  law spectrum is a simple and realistic descriptor of the sea surface, we shall use it.

Assume that the  $K^{-4}$  law is valid for wave numbers in the region of the Bragg condition  $K = 2ka$ . The detailed behavior of the spectrum for wave numbers much smaller than or much larger than the Bragg wave number little affects the scattering. As in section 3.3 we take a low wave number cutoff  $K_0$  corresponding to the spectral peak (cf. eq. (3.18)). The covariance function is very well approximated by eq. (3.19). Now, the second derivatives of the covariance function diverge logarithmically at the origin. As far as the problem of computing the scattering integrals is concerned, we can get around

this mathematical difficulty. We can impose a high-frequency spectral cutoff,  $K = K_\nu$ , say, corresponding to a viscous cutoff. Cutting off the spectrum abruptly this way creates a high frequency oscillation near the origin which is entirely artificial. Rather than doing this, it is simpler (and more sensible) just to "cut out" a small lag region near the origin,  $r \leq r_\nu$ , in which the second derivatives are to be taken as constant. Thus, we define for  $r \leq r_\nu$

$$\nabla^2 R = \frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} = -\sigma_s^2 \quad (7.2)$$

where  $\sigma_s^2$  is the slope variance. In terms of a high frequency spectral cutoff,  $\sigma_s^2$  would be given by

$$\sigma_s^2 = A \ln \frac{K_\nu}{K_0} \quad (7.3)$$

From the definition of the covariance function (eq. (3.19)), viz.,

$$R(r) = \sigma_s^2 + A \frac{r^2}{4} (-1 + \gamma + \ln K_0 r/2) \quad (7.4)$$

it is seen that  $r_\nu$  relates to the cutoff  $K_\nu$  as

$$r_\nu = 2(K_\nu e^\gamma)^{-1} \quad (7.5)$$

We consider only the zeroth order slope terms in the P-integral.

On transforming the Kirchoff integral and the first two terms of the P-integral to polar coordinates  $(r, \phi)$  and integrating over  $\phi$ , we find (using the above definitions):

$$\sigma^{\circ} = K \pm P$$

$$K = 2k^2 \gamma^{-2} \int_0^{\infty} J_0(2k\alpha r) e^{4k^2 \gamma^2 B} r dr$$

$$P = 2k^2 A \int_{r_v}^{\infty} \alpha^2 \ln \frac{r}{r_v} J_0(2k\alpha r) e^{4k^2 \gamma^2 B} r dr \quad (7.6)$$

Figure 7 shows the copolarized returns given by the integrals (7.6). The values of the constants used are:  $A = 4.05 \times 10^{-3}$  (from the Pierson and Moskowitz\* frequency spectrum with  $A = \alpha/2$ ); wind speed  $U = 7.5 \text{ msec}^{-1}$ ;  $k = 2 \text{ cm}^{-1}$  (X-band);  $K_v = 20 \text{ cm}^{-1}$  (a nominal value of the viscous cutoff). In the isotropic case, the perturbation (P-term) is zero at vertical incidence, as we can see by putting  $\alpha = 0$  in (7.6). The failure of the solution at angles approaching  $\theta = 45^\circ$  is seen in the rapid falling away of the horizontally polarized returns as  $K + P$  goes to zero. This failure occurs in the manner described by the linearized solution (eqs. (7.1)) which gives a  $g_{HH}(45^\circ) = 0$ .

A similar treatment can be given to the two-dimensional scattering analog. In the two-dimensional case, the covariance function is given by eq. (3.21), and  $B_{\xi\xi}$  is defined to be

$$B_{\xi\xi} = \begin{cases} 0 & , \quad \xi \leq \xi_v \\ A \ln(\xi/\xi_v) & , \quad \xi \geq \xi_v \end{cases} \quad (7.7)$$

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\*In Neumann and Pierson (1966), pp. 349-352.

where in terms of  $K_\nu$ ,  $\xi_\nu = (K_\nu e^\gamma)^{-1}$ . The first term (zeroth-order in slope) of the P-integral becomes

$$P = 4kA \int_{\xi_\nu}^{\infty} \ln(\xi/\xi_\nu) \cos 2ka\xi e^{4k^2 \gamma^2 B} d\xi \quad (7.8)$$

Fig. 8 shows the copolarized cross-section,  $\sigma^\circ = K \pm P$ , with P given by (7.8) and K given by eq. (3.7). The constants used were  $A = 5 \times 10^{-3}$ ,  $U = 15 \text{ m sec}^{-1}$ ,  $k = 2 \text{ cm}^{-1}$  and  $K_\nu = 20 \text{ cm}^{-1}$ . The values of A, U, and k are the same values used for the Kirchhoff integral shown by curve 'a' in Fig. 3. Again, the solution fails in the neighborhood of  $\theta = 45^\circ$ . The cross-over of the HH and VV curves in the neighborhood of  $\theta = 20^\circ$  is an interesting result of the two-dimensional calculation. This cross-over is clearly evident in the 1.3 GHz backscatter data of McDonald (1956) and the 8.9 GHz data of Daley et al. (1971), which data are reproduced in Fig. 9. On the basis of our theory, it would appear that cross-over in copolarized returns in Fig. 9 is due to anisotropic wave conditions in the high-frequency portion of the wave spectrum.

A calculation of the depolarized return and the higher-order terms in the P and S integrals has been avoided. The kind of approximations we have made to the covariance function and its second derivatives using the  $K^{-4}$  spectrum are not easily extended to the higher-order derivatives. In dealing with third derivatives, one could take the third derivative of the covariance function as it stands with the  $r^{-1}$  singularity at the origin and simply let the singularity be removed through the integration. Fourth derivatives have a singularity proportional to the  $r^{-2}$  which is not removed in the integration.

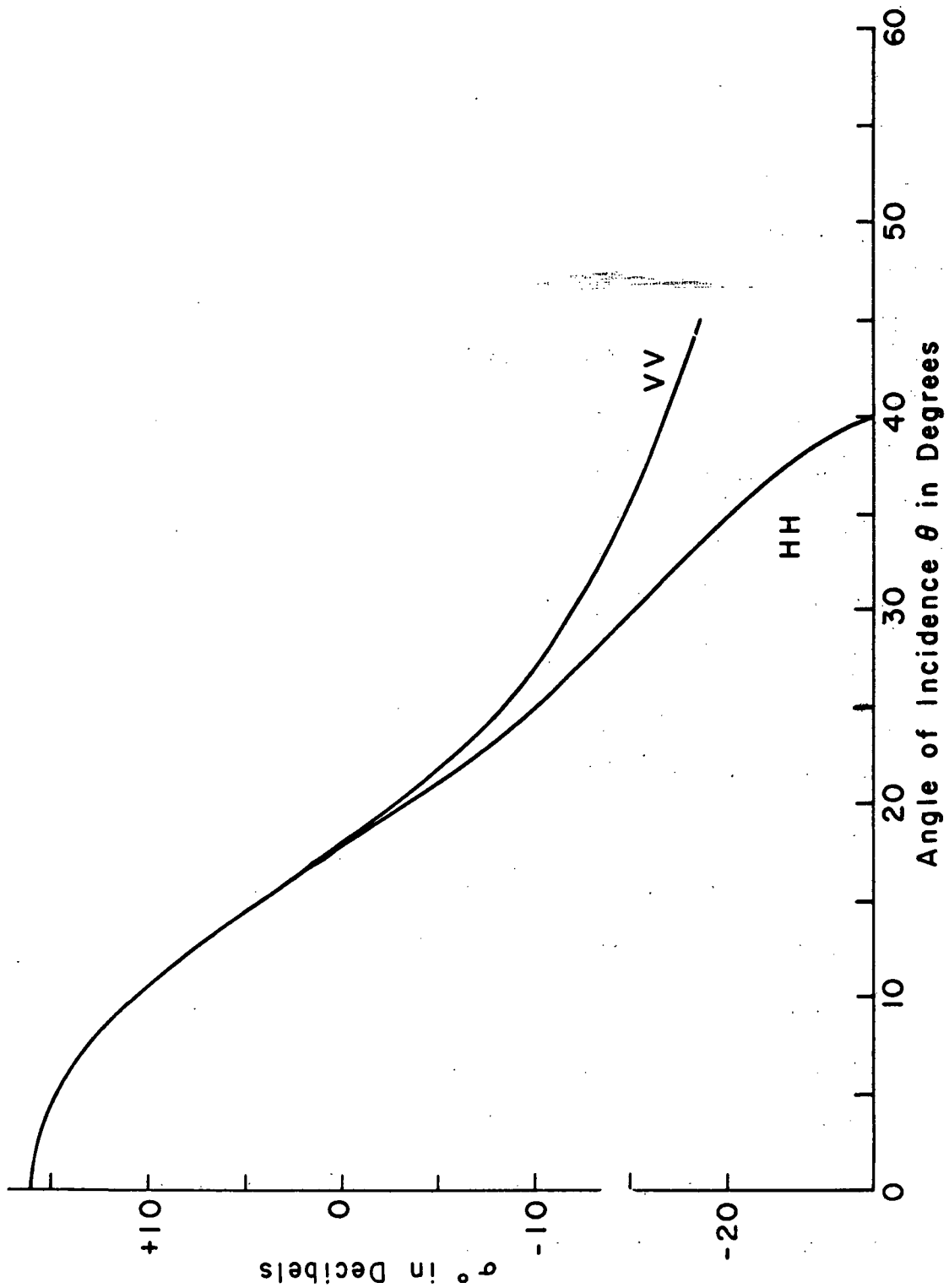


Fig. 7. Three-dimensional corrected Kirchhoff solution for copolarized returns from an isotropic  $K^{-4}$  spectral-law surface.

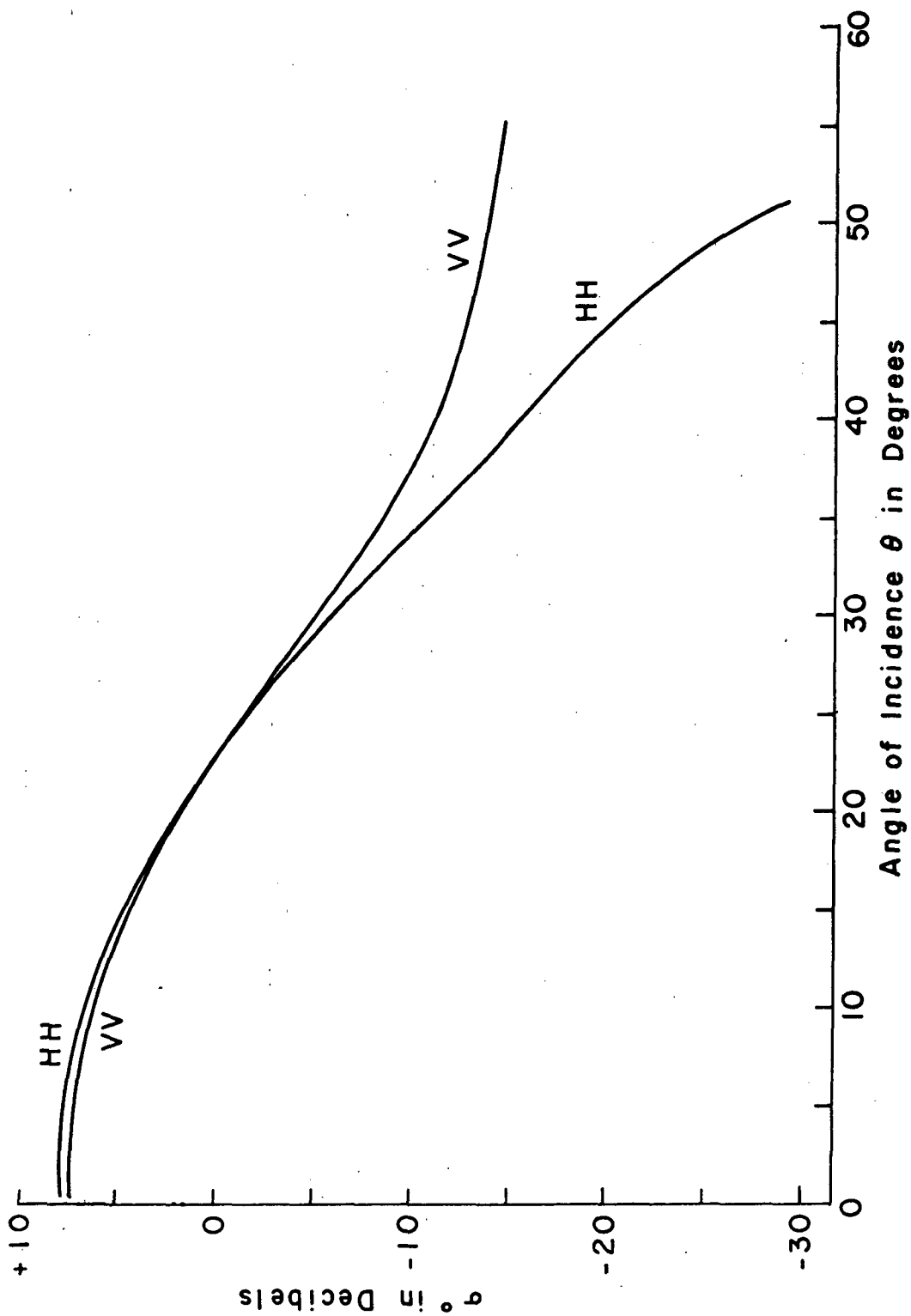


Fig. 8. Two-dimensional corrected Kirchhoff solution for copolarized returns from a  $K^{-3}$  spectral-law surface.

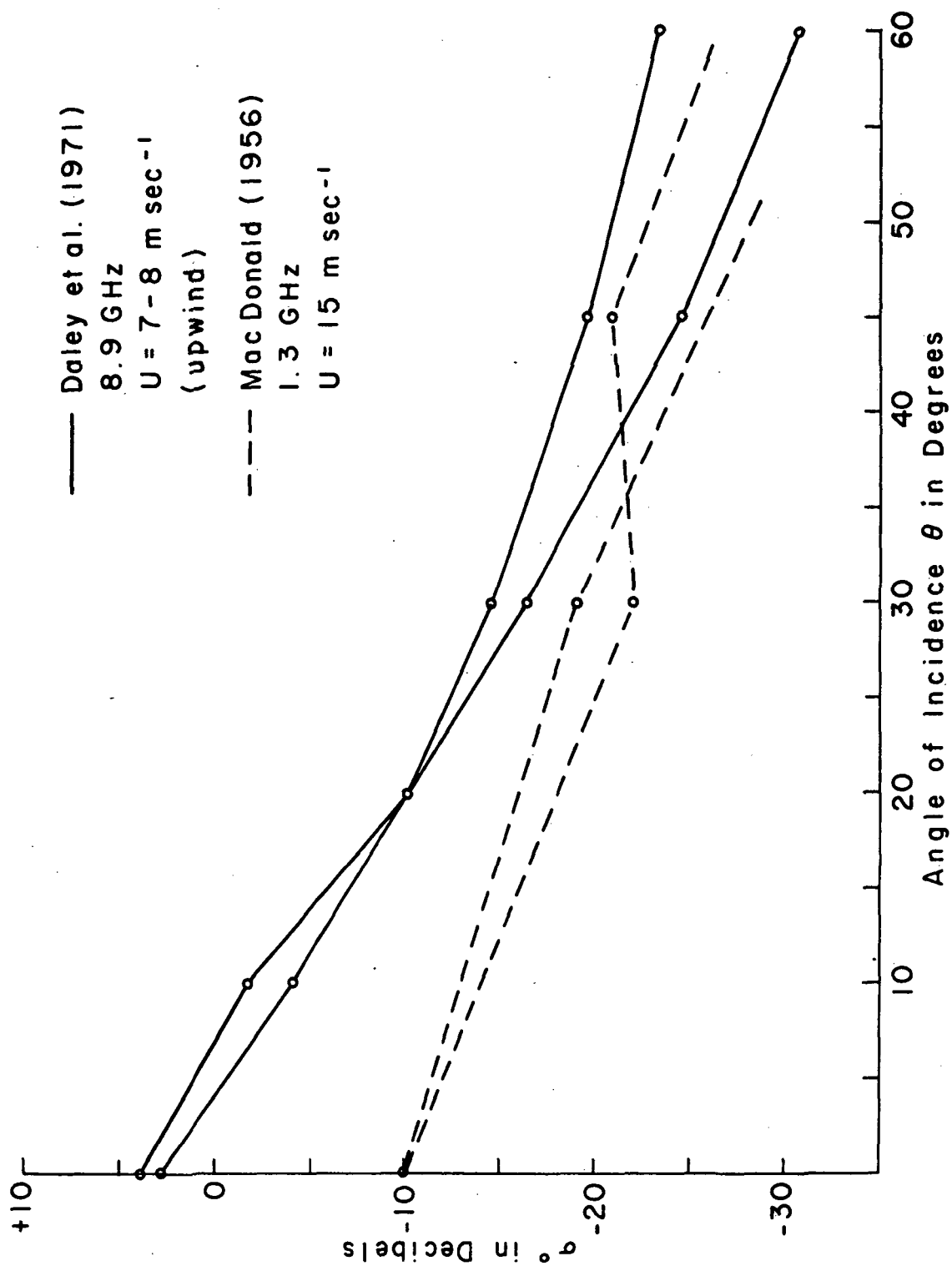


Fig. 9. Ocean backscatter data showing cross-over of copolarized returns.

A detailed analysis of how the scattering integrals might be "best applied" to the radar sea-return problem is outside the scope of this work. For X-band radar sea return, an accurate spectral representation of the high-frequency gravity and capillary waves including a viscous cutoff is one possibility. Then, all required derivatives of the covariance function will exist and the scattering integrals could then be computed. For lower-frequency microwaves, a detailed modelling of the covariance function near the origin is not necessary. But it remains a problem how to control most reasonably the behavior of the higher-order derivatives near the origin. We see that the practical problem of using the scattering integrals is intimately linked with the problem of error in this high-frequency approximation to the scattering problem.



### VIII. Discussion and Conclusions to be Drawn

A few years ago, Prof. R. K. Moore pointed out the need for extending Kirchoff techniques beyond the tangent-plane approximation.\* With the curvature correction to the tangent-plane approximation, this has in a good measure been accomplished.

The sample calculations illustrate the potential strength of this "corrected Kirchoff theory" for predicting radar sea-return from the near-vertical (specular regime) and from the transition region between specular and diffraction regimes. Because of the greater analysis needed to reasonably model the fourth-order mixed partial derivative in the depolarized scattering integral, no sample calculation has been given. A calculation of the depolarized signal should prove most interesting, for it is not masked by the large specular component as the copolarized signals are. As the depolarized return depends essentially on the nonlinear component of the source distribution, this scattering solution is in a peculiarly good position to model depolarization near the vertical. Comparison with the second-order Rayleigh-Rice solution will provide a basis for testing the two solutions over the full range of incidence angles.

On entering the pure diffraction regime the solution deteriorates rapidly due to the over-emphasis on curvature effects and the increasing importance of non-local diffraction processes. It is interesting to note that the tangent-plane approximation alone provides more

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\*In his paper "Scattering from Rough Surfaces" delivered to the Aug. 1969 General Assembly of the URSI, Ottawa, Canada.

reasonable predictions, although it cannot account for the polarization dependence. The excessive splitting of the copolarized returns at larger angles can be controlled by artificial means, and this possibility has not been explored. For example, the wave spectrum could be filtered according to a smoothness criterion in order to remove the high frequency waves which contribute to an intolerably large surface curvature. The derivatives of the covariance function corresponding to the "smoothed" surface could then be used in the scattering integrals. In this way, it would be possible to control the extent of splitting, and to establish a better correspondence with reality. This route is, however, not physically appealing.

Rather than manipulate the solution by artificial means, one should admit the failure of the solution and recognize that at the larger incidence angles small-perturbation methods become applicable. If the Rayleigh-Rice solution shown in Fig. 6 is compared with the corrected Kirchhoff solution, Fig. 8, there is seen a fairly continuous transition between the two solutions in the neighborhood of 45 degrees. The idea of forming a "composite" solution suggests itself. If  $\sigma_1^\circ$  represents the high-frequency cross-section and  $\sigma_2^\circ$  represents the low-frequency cross section,\* then a continuous solution for all incidence angles might be obtained from a weighted addition of the two cross sections: for example,

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\* Here  $\sigma_1^\circ$  might be calculated to first-order in slope, and corrected for finite conductivity by the relationship, eq. (3.9). The small perturbation cross-section might be a first- or second-order Rayleigh-Rice solution for finite conductivity, possibly including the artifice of mean-plane tilting (Valenzuela, 1968; Guinard et al., 1971).

$$\sigma^\circ = W(\theta) \sigma_1^\circ + (1 - W(\theta)) \sigma_2^\circ .$$

The weighting function  $W$  would be near unity for small angles and fall fairly rapidly to near zero at some critical angle in the neighborhood of 45 degrees. The complementary weighting of the two cross-sections guarantees that if both cross-sections were identical (say, were both perfect solutions), then we should have  $\sigma^\circ = \sigma_1^\circ = \sigma_2^\circ$ .

From the standpoint of electromagnetic theory, it appears that between the high-frequency and small-perturbation approximations, the radar sea-return problem is virtually solved. This is, however, within the framework of Gaussian surface statistics. Clearly, we are at a point of theoretical development where a more accurate description of the sea-surface in terms of its statistical properties is needed. It seems to make little sense to calculate the small corrections to the Kirchhoff value near the vertical when the Kirchhoff value itself is likely to be in significant error in its Gaussian form. While the height distribution of ocean-surface gravity waves is to a very good first approximation Gaussian, the skewness of the distribution caused by the nonlinearity of the wave motion becomes increasingly important when considering slope and curvature distributions, and the joint distributions of these variables. Longuet-Higgins (1963) in a remarkable paper has shown how higher-order surface statistics may be derived from the nonlinear hydrodynamical equations of motion for gravity waves. Longuet-Higgins' development in terms of characteristic functions and "cumulants" ties in closely with the methods we have employed to calculate the statistical averages in the scattering integrals, and one is tempted to think of a rather effortless extension

of Kirchoff techniques to the non-Gaussian solution. The problem with a transition to non-Gaussian statistics is that the deviation from normality of gravity waves is quite different from the deviation of capillary waves. For example, gravity waves exhibit a positively skewed distribution function, while the skewness of capillary waves can be negative. For decimeter radars, a modelling of gravity waves alone is possible; but for centimeter radars, both gravity and capillary waves are important, and a simple non-Gaussian model may be near-impossible to develop.

A current research effort by Prof. W. J. Pierson here at New York University is aimed at providing a more detailed description of the high-frequency wave structure in terms of the wave-height spectral density (Pierson et al., 1971). Open-ocean measurements of the high-frequency portion of the wave spectrum (characterized by "chop", "ripples" and capillary waves) are non-existent, although some statistical information is available (e. g., the estimates of slope variance by Cox and Munk (1954)). Hess et al. (1969) have shown that laboratory measurements of wind-generated waves (conducted in "wind-wave tunnels") can provide useful information on the open-ocean wave statistics. Along with certain field observations (e. g., the near-to-shore observations of Leykin and Rosenberg (1970)), laboratory studies are yielding the increased knowledge of the high wave-number portion of the spectrum necessary for more accurate radar sea-return predictions.

Studies concerned with the effect of surface roughness on the natural microwave emission characteristics of the sea require the bistatic scattering cross-section<sup>\*</sup>. In this problem, one is concerned with an absolute power level, and a relative decibel representation of the scattered power pattern is inappropriate. The polarization effects in the near-specular direction will appear small on a logarithmic plot compared to the polarization effects at large angles from the specular direction, although the polarization effects (diffraction effects) in the two regimes are of comparable absolute value. Thus, any effort to account for pure diffraction processes in the bistatic cross-section must model these processes in the near-specular direction, as well as in the pure diffraction regime at angles far from the specular direction. Since this corrected Kirchhoff theory is particularly suited to the near-specular regime, an extension of the scattering integrals to the bistatic case should be considered.

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<sup>\*</sup>For a formulation of the natural emission problem, see Stogryn (1967b).

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## APPENDIX

### Evaluation of an integral

The Fourier integral,

$$\underline{\underline{S}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int \psi^{(1)} \underline{\underline{M}}^{(2)} e^{-i \underline{\ell} \cdot \underline{\xi}} d\xi d\eta \quad (\text{A1})$$

involves three types of integrals, viz.:

$$\int_{-\infty}^{\infty} \int \left\{ \begin{array}{c} \xi^2 \\ \eta^2 \\ \xi\eta \end{array} \right\} \frac{(1 + i k \rho_1)}{\rho_1^3} e^{-i(k\rho_1 + \underline{\ell} \cdot \underline{\xi})} d\xi d\eta. \quad (\text{A2})$$

On transforming to the elliptical coordinates  $(\rho_1, \varphi)$  defined by

$$\begin{aligned} \xi &= \frac{\rho_1}{a} \cos \varphi \\ \eta &= \frac{\rho_1}{b} \sin \varphi \end{aligned} \quad (\text{A3})$$

these integrals become

$$\frac{1}{a b} \int_0^{\infty} \int_0^{2\pi} \left\{ \begin{array}{c} \frac{1}{2} \cos^2 \varphi \\ \frac{1}{2} \sin^2 \varphi \\ \frac{1}{2ab} \sin 2\varphi \end{array} \right\} (1 + i k \rho_1) e^{-i[k\rho_1 + L\rho_1 \cos(\Theta - \varphi)]} d\rho_1 d\varphi \quad (\text{A4})$$

where

$$L = k \left[ \frac{(a - \gamma f_x)^2}{a^2} + \frac{\gamma^2 f_y^2}{b^2} \right]^{\frac{1}{2}} \quad (A5)$$

and

$$\Theta = \tan^{-1} \left[ \frac{-\gamma f_y/b}{(a - \gamma f_x)/a} \right] \quad (A6)$$

Integrating over the polar angle we get

$$\frac{\pi}{ab} \int_0^\infty \left[ \begin{Bmatrix} \frac{1}{a^2} \\ \frac{1}{b^2} \\ 0 \end{Bmatrix} \cdot J_0(L\rho_1) - \begin{Bmatrix} \frac{1}{a^2} \cos 2\Theta \\ -\frac{1}{b^2} \cos 2\Theta \\ \frac{1}{ab} \sin 2\Theta \end{Bmatrix} \cdot J_2(L\rho_1) \right] \cdot (1 + ik\rho_1) e^{-ik\rho_1} d\rho_1 \quad (A7)$$

where  $J_0$  and  $J_2$  are Bessel functions. Evaluation of the radiation integrals, i.e., those integrals involving the factor  $ik\rho_1$  are a bit tricky, and are not to be found in standard tables of Fourier/Bessel transforms. If the substitution  $s = ik$  is made, the integrals become formally Laplace transforms which can be found in tables.

We find that

$$\begin{aligned} \int_0^\infty (1 + ik\rho_1) J_n(L\rho_1) e^{-ik\rho_1} d\rho_1 &= (1 - s \frac{d}{ds}) \int_0^\infty J_n(L\rho_1) e^{-s\rho_1} d\rho_1 \bigg|_{s=ik} \\ &= \begin{cases} -ik^{-1} \nu^{-3} (1 + \nu^2) & \text{for } n = 0 \\ +ik^{-1} \nu^{-3} (1 - \nu^2) & \text{for } n = 2 \end{cases} \end{aligned} \quad (A8)$$

where  $\nu = (1 - L^2/k^2)^{\frac{1}{2}}$ .

The ratio of radiative to inductive contributions is for  $n = 0$ :

$$\frac{\text{Rad}}{\text{Ind}} = \frac{s^2}{s^2 + L^2} \bigg|_{s=ik} = \nu^{-2},$$

and for  $n = 2$ :

$$\frac{\text{Rad}}{\text{Ind}} = \frac{s(2 + s\sqrt{s^2 + L^2})}{\sqrt{s^2 + L^2}} \bigg|_{s=ik} = 2\nu^{-1} + \nu^{-2}. \quad (\text{A9})$$

Substituting (A8) into (A7) yields for the forms (A2):

$$-\frac{2\pi i}{k a b \nu^3} \left\{ \begin{array}{l} (1 - m^2)/a^2 \\ (1 - \ell^2)/b^2 \\ \ell m/a b \end{array} \right\} \quad (\text{A10})$$

where

$$\ell = (a - \gamma_x^f/a)$$

and

$$m = -\gamma_y^f/b.$$



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